

Laplace transzformáció

1. a) $y'' - 4y' + 9y = 0 \quad y(0) = y'(0) = 0$

$$\hookrightarrow \mathcal{L} \rightarrow x^2 \cdot Y(x) - 4x \cdot Y(x) + 9Y(x) = 0 \quad (\text{MÁR AD } 0!)$$

$Y(x) = 0 \rightarrow y(t) = 0$ a megoldásfigur.

b) $y'' - 4y' + 9y = 0 \quad y(0) = 0, y'(0) = 1$

$$\hookrightarrow \mathcal{L} \rightarrow x^2 \cdot Y(x) - x \cdot 0 - 1 - 4x \cdot Y(x) + 9Y(x) = 0$$

$$(x^2 - 4x + 9)Y(x) = 1$$

$$Y(x) = \frac{1}{x^2 - 4x + 9}$$

$$\mathcal{L}^{-1}(Y(x)) = \mathcal{L}^{-1}\left(\frac{1}{(x-2)^2 + 5}\right) = \frac{1}{\sqrt{5}} e^{2t} \sin(\sqrt{5}t)$$

$$y(t) = \frac{1}{\sqrt{5}} \cdot e^{2t} \cdot \sin(\sqrt{5}t)$$

c) $y'' - 4y' + 9y = 0 \quad y(0) = c_1, y'(0) = c_2$

$$\hookrightarrow \mathcal{L} \rightarrow x^2 \cdot Y(x) - x \cdot c_1 - c_2 - 4x \cdot Y(x) + 4c_1 + 9Y(x) = 0$$

$$(x^2 - 4x + 9)Y(x) = x \cdot c_1 + c_2 - 4c_1$$

$$Y(x) = \frac{c_1 (x-2)}{(x-2)^2 + 5} + \frac{(c_2 - 2c_1)}{\sqrt{5}} \frac{\sqrt{5}}{(x-2)^2 + 5}$$

$$\mathcal{L}^{-1}(Y(x)) = c_1 \cdot e^{2t} \cos(\sqrt{5}t) + \frac{c_2 - 2c_1}{\sqrt{5}} e^{2t} \sin(\sqrt{5}t)$$

2. $y'' + 8xy' = 0 \quad y(0) = 4, y'(0) = 0$

$$\hookrightarrow \mathcal{L} \rightarrow x^2 Y(x) - 4x + 8(-x \cdot Y'(x) - Y(x)) = 0$$

$$-8x \cdot Y'(x) + (x^2 - 8)Y(x) = 4x$$

$$Y'(x) = \frac{x^2 - 8}{8x} Y(x) - \frac{1}{2}$$

$$Y_{HA'}(x) : \int \frac{1}{Y(x)} dY = \int \frac{x^2 - 8}{8x} dx = \int \frac{x}{8} - \frac{1}{x} dx$$

$$\ln|Y(x)| = \frac{x^2}{16} - \ln|x| + C$$

$$Y_{HA'}(x) = e^{\frac{x^2}{16}} \cdot \frac{1}{x} \cdot C$$

$$Y_{IP}(x) : C(x) \cdot \frac{1}{x} \cdot e^{\frac{x^2}{16}}$$

alakban

$$C'(x) \cdot \frac{1}{x} \cdot e^{\frac{x^2}{16}} = -\frac{1}{2}$$

$$C(x) = \int -\frac{1}{2} x \cdot e^{\frac{-x^2}{16}} dx = -\frac{1}{2} \cdot -8 \int -\frac{1}{8} x \cdot e^{\frac{-x^2}{16}} dx = 4 \cdot e^{\frac{-x^2}{16}}$$

$$Y_{IP}(x) = 4 \cdot e^{\frac{-x^2}{16}} \cdot \frac{1}{x} \cdot e^{\frac{x^2}{16}} = \frac{4}{x}$$

$$Y(x) = \frac{4}{x}$$

$$Y_{IA'} = C \cdot \frac{1}{x} \cdot e^{\frac{x^2}{16}} + \frac{4}{x}$$

ahol $\lim_{x \rightarrow \infty} Y_{IA'}(x) = 0 \Rightarrow C=0$

$$y(t) = 4$$