Bsc/A3/Gé/2.HF/2011 őszi félév (V. Nagy)

Leadási határidő: a 2. ZH hetének gyakorlatán (12. hét) a gyak. vezér részére (olvashatóan, kézzel írva!)

1. Milyen β-ra lesz egzakt a diff. egyenlet? Ekkor oldja meg!

$$\left(\beta x y^2 e^{x^2} + 3x^2\right) \mathrm{d}\, x + \left(\beta y \cdot e^{x^2} + \beta \sin y \cos y\right) \mathrm{d}\, y = 0,\ y(2) = 0$$

2. Oldja meg!

   (a) $e^x y^2 - 2xy + (2ye^x - x^2)y' = 0,\ y(0) = 2$

   (b) $(y\cos^2 x - \sin x)\,\mathrm{d}\, y - y\cos x(y\sin x + 1)\,\mathrm{d}\, x = 0$

   (c) $y' e^{-x}\cos y = e^{-x}\sin y - \cos x$

3. Oldja meg!

   (a) $y' = \frac{x^2}{y^3},\ y' = 2^{x-3y},\ y' = x^2 \cdot y^2$

   (b) $y' = \frac{(x^2-2x)e^{-x}}{(2y+1)\sin 2y},\ y' = \frac{(2x+3)\cos 3x}{(2y^2+3y)\ln^2 y},\ y' = \frac{x^5+3x^2+1}{(e^y \cdot \sin 2y)(x^2+1)}$

4. Oldja meg!

   (a) $(5x - 7y) - (x + y)y' = 0,\ y(1) = 1$

   (b) $y' = \frac{y}{x}\sin\frac{y}{x},\ y(\frac{1}{2}) = \frac{\pi}{4}$

   (c) $y' = \frac{y}{x}\left(1 + \frac{y-6x}{2y+6x}\right),\ y(1) = 2$

   (d) $(x + y)^2 y' = x^2 - 2xy + 5y^2,\ y(1) = 1$

5. Oldja meg!

   (a) $y' = \sqrt{2x + 3y - 1} + 4x + 6y + 3$

   (b) $y' = 4 + \cos(-5x + 2y + 3),\ y(1) = 1$

   (c) $y' = e^{2y-6x+1} + 3,\ y(0) = 0$

6. Oldja meg!

   (a) $y' + \frac{y}{x} = \sqrt{x},\ y(1) = 1$

   (b) $y'x - \frac{y}{x+1} = x$

   (c) $y'\sin x + \cos x = 1 + y$

   (d) $y'\cos x + y\sin x = 1,\ y(0) = 1$

   (e) $y' + \frac{y}{x} + e^x = 0,\ y(1) = 0$

7. (a) $y(y - 1)y'' + (y')^2 = 0$

   (b) $(1 + x^2)y'' + (y')^2 + 1 = 0$

   (c) $y \cdot y'' + (y')^2 = 1,\ y(0) = 1, y'(0) = 0$

8. (a) $2y'' - y' - 3y = 2\sin x + \frac{5}{e^x}$

   (b) $y'' - 4y' + 4y = e^{3x} + 2e^{2x},\ y(0) = 1, y'(0) = 1$

   (c) $y'' - y' - 6y = \sin x + 2\cos 3x$

   (d) $y'' - 2y' + 2y = 4e^{2x} + 2$

9. (a) $\underline{\dot{x}} = \begin{pmatrix} 6 & -1 \\ 4 & 2 \end{pmatrix} \underline{x} + e^{4t} \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

   (b) $\dot{x} = x + 2y + 1$
   $\dot{y} = x + y + t$

   (c) $\dot{x} = -7x + y + t,\ x(0) = 3$
   $\dot{y} = -2x - 5y + 2,\ y(0) = -5$

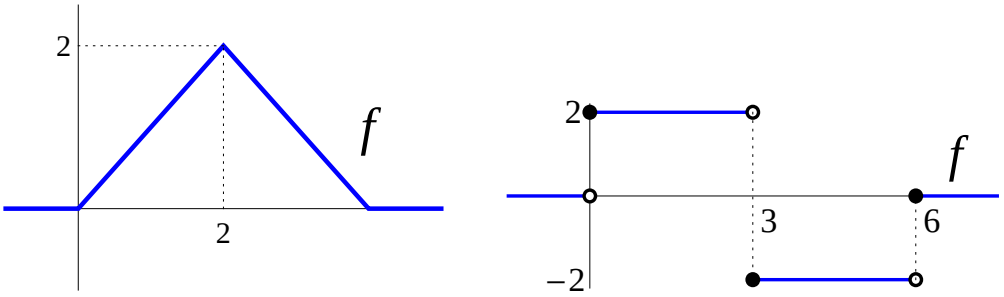
10. $\mathcal{L}(f(t))(s) = ?$

   (a) $t \sinh 3t$

   (b) $te^t + \cosh t$

   (c) $t^2 \cos t$

   (d) $\int_0^t \tau^2 e^{-\tau} \,\mathrm{d}\,\tau$



(e) (f)

10. ábra.

11. $\mathcal{L}(f)(s) = \ln \frac{s^2+1}{(s-1)^2},\ f(t) = ?$

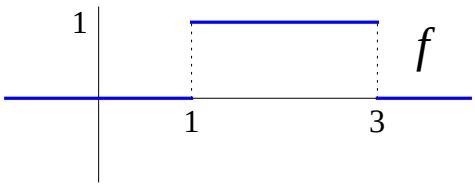
12. $f(t) = ?,\ \mathcal{L}(f(t))(s) = ?$

   (a) $1 * \cos t$

   (b) $e^t * t$

   (c) $\sin^2 t * t$

13. (a) $\ddot{x}(t) - 3\dot{x}(t) + 2x(t) = f(t),\ x(0) = \dot{x}(0) = 1$



13. ábra.

   (b) $x^{(4)}(t) + 2\ddot{x}(t) + x(t) = \cos t,\ x(0) = \dot{x}(0) = \ddot{x}(0) = \dddot{x}(0) = 0$

   (c) $t\ddot{x}(t) + (1-t)\dot{x}(t) + x(t) = 0,\ x(0) = 1, \dot{x}(0) = -1$

14. (a) $1 - \cos t = \int_0^t y(\tau) \sinh(t-\tau) \,\mathrm{d}\,\tau,\ y(t) = ?$

   (b) $\dot{y}(t) - 2\int_0^t y(\tau) e^{t-\tau} \,\mathrm{d}\,\tau,\ y(0) = 1, y(t) = ?$

   (c) $y(t) = \sin t + \int_0^t y(\tau) e^{t-\tau} \,\mathrm{d}\,\tau,\ y(t) = ?$