A2 Gyakorlat
Műszaki Menedzser szakos hallgatóknak

9. hét - Mátrixok sajátértékei, sajátvektorai - Megoldások

Feladatok:

1. Feladat.

a) $\lambda_1 = 8$, $\mathbf{s_1} = (2t, t)$; $\lambda_2 = -1$, $\mathbf{s_2} = (-t, t)$

b) $\lambda_1 = 3$, $\mathbf{s_1} = (2t, 2t, t)$; $\lambda_2 = 2$, $\mathbf{s_2} = (t, t, 0)$; $\lambda_3 = 1$, $\mathbf{s_3} = (0, -2t, t)$

c) $\lambda_1 = 4$, $\mathbf{s_1} = (0, 0, t)$; $\lambda_2 = \frac{1+\sqrt{3}i}{2}$, $\mathbf{s_2} = ((-1 + 2\sqrt{3}i)t, (-3, 5 - 4\sqrt{3}i)t, t)$; $\lambda_3 = \frac{1-\sqrt{3}i}{2}$, $\mathbf{s_3} = ((-1 + 2\sqrt{3}i)t, (-3, 5 + 4\sqrt{3}i)t, t)$

d) $\lambda_1 = 1$, $\mathbf{s_1} = (t, t, t)$; $\lambda_2 = \frac{-1+\sqrt{3}i}{2}$, $\mathbf{s_2} = (\frac{5-\sqrt{3}i}{6}t, \frac{5+\sqrt{3}i}{6}t, t)$; $\lambda_3 = \frac{-1-\sqrt{3}i}{2}$, $\mathbf{s_3} = (\frac{5+\sqrt{3}i}{6}t, \frac{5-\sqrt{3}i}{6}t, t)$

e) $\lambda_1 = 6$, $\mathbf{s_1} = (3t, 4t)$; $\lambda_2 = -1$, $\mathbf{s_2} = (-t, t)$

f) $\lambda_1 = 7$, $\mathbf{s_1} = (t, t)$; $\lambda_2 = -2$, $\mathbf{s_2} = (-4t, 5t)$

2. Feladat.

a) $\begin{pmatrix} \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} \\ 0 & -1 & 0 \\ \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{pmatrix}$, $\lambda_1 = -1$, $\{t(0, 1, 0)|t \in \mathbb{R}\}$ - a forgástengely;

$\quad\quad\quad\quad \lambda_2 = \frac{1+i}{\sqrt{2}}; \lambda_3 = \frac{1-i}{\sqrt{2}}$, $\{t(i, 0, 1) + s(-i, 0, 1)|t, s \in \mathbb{R}\}$ - a tükrözési sík;

b) $\begin{pmatrix} 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 1 & 0 & 0 \end{pmatrix}$, $\lambda_1 = -1$, $\{t(-1, 0, 1)|t \in \mathbb{R}\}$ - a tükrözési síkra merőleges egyenes;

$\quad\quad\quad\quad \lambda_2 = \lambda_3 = 1$, $\{t(1, 0, 1) + s(0, 1, 0)|t, s \in \mathbb{R}\}$ - a tükrözési sík;

c) $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \end{pmatrix}$, $\lambda_1 = 1$, $\{t(0, 1, 0)|t \in \mathbb{R}\}$ - a forgástengely;

$\quad\quad\quad\quad \lambda_2 = \frac{\sqrt{3}}{2}$, $\{t(0, 0, 1)|t \in \mathbb{R}\}$ - a z-tegely; $\quad\quad \lambda_3 = 0$, $\{t(-\sqrt{3}, 0, 1)|t \in \mathbb{R}\}$

d) $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$, $\lambda_1 = 2$, $\{t(1, 0, 0)|t \in \mathbb{R}\}$ - a forgástengely;

$\quad\quad\quad\quad \lambda_2 = i; \lambda_3 = -i$, $\{t(0, i, 1) + s(0, -i, 1)|t, s \in \mathbb{R}\}$ - a forgástengelyre merőleges sík

3. Feladat.

a) $\begin{pmatrix} 12 & 0 \\ 0 & 2 \end{pmatrix}$, $\mathbf{b_1} = \frac{1}{\sqrt{10}}(1,3);\ \mathbf{b_2} = \frac{1}{\sqrt{10}}(-3,1);$

b) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\mathbf{b_1} = \frac{1}{\sqrt{2}}(1,1);\ \mathbf{b_2} = \frac{1}{\sqrt{2}}(-1,1);$

c) $\begin{pmatrix} 6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -2 \end{pmatrix}$, $\mathbf{b_1} = \frac{1}{\sqrt{6}}(1,2,1);\ \mathbf{b_2} = \frac{1}{\sqrt{3}}(1,-1,1);\ \mathbf{b_3} = \frac{1}{\sqrt{2}}(-1,0,1);$

d) $\begin{pmatrix} \frac{1+\sqrt{33}}{2} & 0 & 0 \\ 0 & \frac{1-\sqrt{33}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $\mathbf{b_1} = (\frac{1+\sqrt{33}}{4},-1,1);\ \mathbf{b_2} = (\frac{1-\sqrt{33}}{4},-1,1);\ \mathbf{b_3} = (0,1,1);$

e) $\begin{pmatrix} 6 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{pmatrix}$, $\mathbf{b_1} = \frac{1}{3}(-2,-1,2);\ \mathbf{b_2} = \frac{1}{\sqrt{2}}(1,0,1);\ \mathbf{b_3} = \frac{1}{\sqrt{5}}(-1,2,0);$

f) $\begin{pmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $\mathbf{b_1} = \frac{1}{\sqrt{2}}(-1,0,1);\ \mathbf{b_2} = \frac{1}{\sqrt{5}}(-2,1,0);\ \mathbf{b_3} = \frac{1}{\sqrt{6}}(1,2,1);$

4. Feladat.

a) $9X^2 + Y^2 = 9$
b) $(5 + \sqrt{10})X^2 + (5 - \sqrt{10})Y^2 = -14$
c) $9X^2 - Y^2 = -11$