A2 Gyakorlat
Műszaki Menedzser szakos hallgatóknak

5. hét - Sorfejtés: hatványsorok, Taylor-sorok, Fourier-sorok - Megoldások

Feladatok:

1. Feladat.

a) $\cos(5x) = \sum_{n=0}^{\infty} \frac{(-25)^n}{(2n)!} x^{2n}, \forall x \in \mathbb{R}$

b) $\cos(\sqrt{x}) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{n}, \forall x \in [0, \infty)$

c) $\operatorname{sh}\left(\frac{x}{3}\right) = \sum_{n=0}^{\infty} \frac{1}{3^{2n+1}(2n+1)!} x^{2n+1}, \forall x \in \mathbb{R}$

d) $\sin(x - \frac{\pi}{4}) = \sum_{n=0}^{\infty} \frac{(-1)^(n+1)}{\sqrt{2}(2n)!} x^{2n} + \frac{(-1)^n}{\sqrt{2}(2n+1)!} x^{2n+1}, \forall x \in \mathbb{R}$

e) $e^{-x^2} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^{2n}, \forall x \in \mathbb{R}$

f) $\sqrt[3]{e^x} = \sum_{n=0}^{\infty} \frac{1}{3^n n!} x^n, \forall x \in \mathbb{R}$

g) $(1+x)^3 = \sum_{n=0}^{3} \binom{3}{n} x^n, \forall x \in \mathbb{R}$

h) $(1+x)^{-\frac{1}{3}} = \sum_{n=0}^{\infty} \binom{-\frac{1}{3}}{n} x^n, \forall |x| < 1$

i) $\frac{x}{4+x^2} = \sum_{n=0}^{\infty} \frac{(-1)^n}{4^n} x^{2n+1}, \forall |x| < 4$

j) $\sqrt[3]{8+x} = \sum_{n=0}^{\infty} \binom{\frac{1}{3}}{n} \frac{x^n}{2^{3n-1}}, \forall |x| < 8$

k) $\ln(1-x^2) = \sum_{n=0}^{\infty} \frac{-1}{n} x^{2n}, |x| < 1$

l) $\operatorname{arctg}(2x) = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot 2^{2n+1}}{2n+1} x^{2n+1}, \forall x \in \mathbb{R}$

2. Feladat.

a) $\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \left(x - \frac{\pi}{2}\right)^{2n}, \forall x \in \mathbb{R}$

b) $\sin(x) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{2} \cdot n!} \left(x - \frac{\pi}{4}\right)^{2n}, \forall x \in \mathbb{R}$

c) $\ln(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} (x-1)^n, |x-1| < 1$

d) $\ln(x) = 1 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{e^n n} (x-e)^n, |x-e| < e$

e) $2x^3 - x = -\frac{1}{4} + \frac{1}{2}\left(x - \frac{1}{2}\right) + 3\left(x - \frac{1}{2}\right)^2 + 2\left(x - \frac{1}{2}\right)^3, \forall x \in \mathbb{R}$

f) $\frac{x+1}{x+3} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2^n}(x+1)^n, |x+1| < 2$

$\frac{x+1}{x+3} = \frac{c+1}{c+3} + \sum_{n=1}^{\infty} \frac{2 \cdot (-1)^{n+1}}{(c+3)^{n+1}}(x-c)^n, |x-c| < |c+3|$

g) $\sqrt{x+1} = \sum_{n=0}^{\infty} \binom{\frac{1}{2}}{n} \frac{(-1)^n}{2^{2n-1}}(x-3)^n, |x-3| < 4$

h) $\frac{1}{x+1} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(c+1)^{n+1}}(x-c)^n, |x-c| < |c+1|$

3. Feladat.

a) $\frac{1}{4} - \frac{1}{3 \cdot 4^3} \approx 0.2448$

b) $\frac{1}{2} + \frac{1}{48} \approx 0.5208$

c) $1 + \frac{2}{300} \approx 1.0067$

d) $3\left(1 - \frac{1}{4 \cdot 81}\right) \approx 2.9907$

4. Feladat.

a) ≈ 0.4166667

b) ≈ 0.188286

c) ≈ 0.484375

5. Feladat.

a) $f(x) = \frac{\pi}{2} + \sum_{k=1}^{\infty} \frac{1-(-1)^k}{k} \sin(kx)$

b) $f(x) = \frac{\pi}{4} + \sum_{k=1}^{\infty} \frac{-1+(-1)^k}{k^2\pi} \cos(kx) + \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} \sin(kx)$

c) $f(x) = \frac{\pi}{2} - 1 + \sum_{k=1}^{\infty} \frac{-1+(-1)^k}{k^2\pi} \cos(kx) + \sum_{k=1}^{\infty} \left(\frac{1-(-1)^k}{k\pi} + \frac{(-1)^{k+1}}{k}\right) \sin(kx)$

d) $f(x) = \frac{\pi^2}{6} + \sum_{k=1}^{\infty} \frac{2 \cdot (-1)^n}{k^2} \cos(kx) + \sum_{k=1}^{\infty} \frac{-2+(-1)^k(2-k^2\pi^2)}{k^3\pi} \sin(kx)$

e) $f(x) = \frac{1}{\pi} + \frac{1}{2}\sin(x) + \sum_{k=1}^{\infty} \frac{1}{1-4k^2} \cos(2kx)$

f) $f(x) = \frac{1}{2} + \sum_{k=1}^{\infty} \frac{-1+(-1)^k}{2k^2\pi^2} \cos(2k\pi x) + \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2k\pi} \sin(2k\pi x)$

g) $f(x) = \frac{1}{3} + \frac{4}{\pi^2} \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2} \cos(k\pi x)$