Mathematics A1a practice

Academic year 2016 autumn

Practive examples for November 24

The second midterm will be on 24th November.

1.
$$\int \frac{\sqrt[3]{x^2}\cdot\sqrt{x^3}}{\sqrt[4]{x}}\,\mathrm{d}x = \int \frac{x^{\frac{2}{3}}\cdot x^{\frac{3}{2}}}{x^{\frac{1}{4}}}\,\mathrm{d}x = \int x^{\frac{2}{3}+\frac{3}{2}-\frac{1}{4}}\,\mathrm{d}x = \int x^{\frac{23}{12}}\,\mathrm{d}x = \frac{x^{\frac{35}{12}}}{\frac{35}{12}} + c$$

2.
$$\int \frac{(2x+1)^2}{x}\,\mathrm{d}x = \int \frac{4x^2+4x+1}{x}\,\mathrm{d}x = \int 4x+4+\frac{1}{x}\,\mathrm{d}x = 2x^2+4x+\ln(x)+c$$

3.
$$\int (2x-3)^{10}\,\mathrm{d}x = \frac{1}{2}\cdot\frac{(2x-3)^{11}}{11}+c$$

use linear substitution.

4.
$$\int \sqrt[3]{(2x+1)^2}\,\mathrm{d}x = \int (2x+1)^{\frac{2}{3}}\,\mathrm{d}x = \frac{1}{2}\cdot\frac{(2x+1)^{\frac{2}{3}+1}}{\frac{2}{3}+1}+c = \frac{3}{10}\cdot(2x+1)^{\frac{5}{3}}+c$$

5. linear substitution
$$\int \sin(3x+2)\,\mathrm{d}x = -\frac{1}{3}\cos(3x+2)+c$$

6.
$$\int \cos^2 x\,\mathrm{d}x = \int \frac{1+\cos 2x}{2}\,\mathrm{d}x = \int \frac{1}{2}+\frac{1}{2}\cdot\cos 2x\,\mathrm{d}x = \frac{1}{2}\cdot x+\frac{1}{4}\cos(2x)+c$$

7.
$$\int 2\sinh(4x+2)\,\mathrm{d}x = 2\cdot\frac{1}{4}\cosh(4x+2)+c$$

8.
$$\int 3^x\cdot 2^{2x}\,\mathrm{d}x = \int e^{\ln(3)\cdot x}\cdot e^{\ln(2)\cdot 2\cdot x}\,\mathrm{d}x = \int e^{(\ln(3)+\ln(2)\cdot 2)\cdot x}\,\mathrm{d}x = \frac{1}{\ln(3)+\ln(2)\cdot 2}\cdot\underbrace{e^{(\ln(3)+\ln(2)\cdot 2)\cdot x}}_{3^x\cdot 2^{2x}}+c$$

9.
$$\int \frac{3\cdot e^x}{2\cdot e^{3x}}\,\mathrm{d}x = \int \frac{3}{2}\cdot e^{-2x}\,\mathrm{d}x = \frac{3}{2}\cdot\frac{1}{-2}\cdot e^{-2x}+c$$

10.
$$\int \frac{1}{2x+1}\,\mathrm{d}x = \frac{1}{2}\ln(2x+1)+c$$

11.
$$\int \frac{3}{2+x^2}\,\mathrm{d}x = \frac{3}{2}\cdot\int \frac{1}{1+\frac{1}{2}\cdot x^2}\,\mathrm{d}x = \frac{3}{2}\cdot\int \frac{1}{1+\left(\frac{1}{\sqrt{2}}\cdot x\right)^2}\,\mathrm{d}x = \underbrace{\frac{3}{2}\cdot\sqrt{2}}_{\frac{3}{\sqrt{2}}}\cdot\arctan\left(\frac{1}{\sqrt{2}}\cdot x\right)+c$$

12.
$$\int \frac{2}{2+3x^2}\,\mathrm{d}x = \int \frac{1}{1+\frac{3}{2}x^2}\,\mathrm{d}x = \int \frac{1}{1+\left(\sqrt{\frac{3}{2}}\cdot x\right)^2}\,\mathrm{d}x = \frac{1}{\sqrt{\frac{3}{2}}}\cdot\arctan\left(\sqrt{\frac{3}{2}}\cdot x\right)+c$$

13.

$$\int \frac{2}{\sqrt{4-x^2}}\,\mathrm{d}x = \not{2}\cdot \int \frac{1}{\sqrt{\not{4}\cdot\left[1-\left(\frac{1}{2}\cdot x\right)^2\right]}}\,\mathrm{d}x = \underbrace{\frac{1}{\frac{1}{2}}}_{2}\cdot \arcsin\left(\frac{1}{2}\cdot x\right)+c$$

14.

$$\int \frac{2}{2-x^2}\,\mathrm{d}x = \int \frac{2}{(\sqrt{2}-x)(\sqrt{2}+x)}\,\mathrm{d}x$$

We do partional fraction decomposition.

$$\frac{2}{(\sqrt{2}-x)(\sqrt{2}+x)} = \frac{A}{\sqrt{2}-x}+\frac{B}{\sqrt{2}+x}$$
$$2 = A\cdot(\sqrt{2}+x)+B\cdot(\sqrt{2}-x)$$
$$2 = A\cdot\sqrt{2}+A\cdot x+B\cdot\sqrt{2}-B\cdot x$$
$$2 = (A-B)\cdot x + A\cdot\sqrt{2}+B\cdot\sqrt{2}$$
$$0\cdot x+2 = \underbrace{(A-B)}_{0}\cdot x + \underbrace{A\cdot\sqrt{2}+B\cdot\sqrt{2}}_{2}$$
$$\Downarrow$$
$$A = B = \frac{1}{\sqrt{2}}$$

So

$$\int \frac{2}{\sqrt{4-x^2}}\,\mathrm{d}x = \int \frac{\frac{1}{\sqrt{2}}}{\sqrt{2}-x}\,\mathrm{d}x + \int \frac{\frac{1}{\sqrt{2}}}{\sqrt{2}+x}\,\mathrm{d}x = \frac{1}{\sqrt{2}}\cdot\Big(-\ln(\sqrt{2}-x)+\ln(\sqrt{2}+x)\Big)+c$$

15.

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,\mathrm{d}x$$

Using $u=\sqrt{x}$ and $\mathrm{d}u = \frac{1}{2\sqrt{x}}\,\mathrm{d}x$

$$= 2\cdot\int e^u\cdot\underbrace{\frac{1}{2\sqrt{x}}\,\mathrm{d}x}_{\mathrm{d}u} = 2\cdot\int e^u\,\mathrm{d}u = 2\cdot e^u+c = 2\cdot e^{\sqrt{x}}+c$$

16.

$$\int \frac{e^{4x}}{e^x+1}\,\mathrm{d}x$$

Using $u=e^x$ and $\frac{\mathrm{d}u}{\mathrm{d}x} = e^x = u$ we also have that $\frac{1}{u}\,\mathrm{d}u = \mathrm{d}x$.

$$\int \frac{u^4}{u+1}\cdot\frac{1}{u}\cdot\,\mathrm{d}u = \int \frac{u^3}{u+1}\,\mathrm{d}u$$

We do polynom division

$$\begin{array}{llll}
u^3 & & & : (u+1) = u^2-u+1 \\
-(u^3 & +u^2) & & \\
 & -u^2 & & \\
 & -(-u^2 & -u) & \\
 & & u & \\
 & & -(u & +1) \\
 & & & -1
\end{array}$$

Which means that $\frac{u^3}{u+1} = u^2-u+1-\frac{1}{u+1}$

$$\int \frac{u^3}{u+1}\,\mathrm{d}u = \int u^2-u+1-\frac{1}{u+1}\,\mathrm{d}u = \frac{u^3}{3}-\frac{u^2}{2}+u-\ln(u+1)+c = \frac{e^{3x}}{3}-\frac{e^{2x}}{2}+e^x-\ln(e^x+1)+c$$