Math A1a - Calculus
Civil Engineering
Final exam (Autumn 2016)

Name (Print): _____
Neptun Code: _____
Time Limit: 100 Minutes

This exam contains 13 pages (including this cover page) and 6 questions. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your Neptun code on the top of every page, in case the pages become separated.

You are required to show your work on each question. You can collect points from any questions, no restriction on your choice. You can collect at most 100 points in the whole exam. The following rules apply:

- You must not use your books, notes, or any other papers. You can only use blue or black pen, answers written with red pen, green pen or pencil will not be considered.
- You may use your calculator but you are not allowed to use any other device, e.g. your phone.
- Organize your work, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- Mysterious or unsupported answers will not receive full credit. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- You have to work alone, any communications with the others will cause termination of your exam immediately.
- If you leave the room during the exam for any reason, then your exam will be terminated.
- If you need more space, ask the examiner to provide some blank papers.

Question	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
Total:	100	

Do not write in the table to the right.

With best wishes,
Dr. Mohamed Khaled and Gábor Borbély
January 10, 2017

Question 1 ..... *20 points*

(a) [5 points] Find all real and complex solutions of $z^5 - 16z = 0$.

(b) [5 points] Find the square roots of the complex number $(1 + i\sqrt{3})$.

(c) [5 points] Is the function $f(x) = \frac{x}{x-1}$ invertible? If yes, then find $f^{-1}(x)$.

(d) [5 points] Suppose that we know that f is continuous and differentiable on $\mathbb{R}$. Suppose also that we know that f has two roots. Show that f' must have at least one root.

Question 2 ..... *20 points*

(a) [8 points] Sketch the graph of the function $y = x^3 + 2x^2$.

(b) [5 points] Given that $\lim_{x\to 0} \frac{1}{x}$ does not exist. $\lim_{x\to 0} \left(\frac{1}{x} - x\right)$ also does not exist. (Hint: $\frac{1}{x} = \frac{1}{x} - x + x$ is true for every $x \neq 0$)

(c) [7 points] Let $a \neq 1$ be a real number. Show that

$$\lim_{n\to\infty} \frac{1}{n^{a+1}}(1^a + 2^a + \cdots + n^a) = \frac{1}{a+1}$$

Question 3 ..... *20 points*

(a) [5 points] State the Mean Value Theorem for integrals.

(b) Evaluate the following integrals:

i. [5 points] $\displaystyle\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,\mathrm{d}x$

ii. [5 points] $\int \sin^2 5x \, \mathrm{d}x$

iii. [5 points] $\int x\sqrt{1-x^2} \arcsin x \, \mathrm{d}x$

Question 4 ..... *20 points*

(a) [5 points] Prove that

$$\int x^m (\ln x)^n \, \mathrm{d}x = \frac{x^{m+1}(\ln x)^n}{m+1} - \frac{n}{m+1} \int x^m (\ln x)^{n-1} \, \mathrm{d}x, \ m \neq 1. \tag{1}$$

(b) [5 points] Use equation (1) above to evaluate $\int (\ln x)^2 \, \mathrm{d}x$.

(c) Find each of the following limits or show that it does not exist:

i. [5 points] $\lim_{x \to 0} \dfrac{\sqrt{x^2 + 9x + 9} - 3}{x}$

ii. [5 points] $\lim_{x \to \infty} \dfrac{\sqrt{x^2 + 1}}{\ln x}$

Question 5 ..... *20 points*

(a) [5 points] Find the relative position, the angle and the intersection (if any) of the line $(3, 1, 3) + (1, 0, 1) \cdot t$ and the plane $x - y + z = 2$.

(b) [5 points] How do you determine the area of a parallelogram given with its vertices (2D)? Suppose that you are given the coordinates of each vertex.

(c) [5 points] $\lim_{n\to\infty} \dfrac{\sqrt{n^2+n+1}}{(n+1)^2-\sqrt{n+1}} =?$

(d) [5 points] Give an example of a sequence which is bounded but not convergent.

Question 6 ..... *20 points*

(a) [5 points] Differentiate the following function $y = \frac{e^x}{\cos x}$.

(b) [5 points] Determine the tangent and normal of the graph $y = \sqrt[3]{x^2 - 1}$ at $x_0 = 2$.

(c) [5 points] Determine the length of the graph $\sqrt{1-x^2}$ with indefinite limits (indefinite integral).

(d) [5 points] Is the following improper integral finite or infinite?

$$\int_1^\infty \frac{\sqrt{x^2-1}}{x^2+1}\,\mathrm{d}x$$