Math A1a - Calculus **Name (Print):** _____
Civil Engineering **Neptun Code:** _____
1st (rep.) mid-term (Autumn 2016) **Time Limit:** 90 Minutes

This exam contains 11 pages (including this cover page) and 4 questions. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your neptun code on the top of every page, in case the pages become separated.

You are required to show your work on each question. You can collect points from any question, no restriction on your choice. You can collect at most 100 points in the whole exam.

The following rules apply:

- You must **not** use your books, notes, or any other papers. You can **only** use blue or black pen, answers written with red pen, green pen or pencil will not be considered.
- You may use your calculator but you are **not** allowed to use any other device, e.g. **YOUR PHONE**.
- Organize your work, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- Mysterious or unsupported answers will not receive full credit. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- You have to work alone, any communications with the others will cause termination of your exam immediatly.
- If you leave the room during the exam for any reason, then your exam will be terminated.
- If you need more space, ask the examiner to provide some blank papers.

Question	Points	Score
1	30	
2	30	
3	33	
4	22	
Total:	100	

Do not write in the table on the right.

With best wishes,
Dr. Mohamed Khaled and Gábor Borbély
December 2, 2016

Question 1 ..... *30 points*

(a) [5 points] Generate the truth table for the following formula, then state whether it is tautology, contradiction or satisfiable.

$$(p \to q) \vee (\neg p \to r).$$

(b) [5 points] Prove that it is impossible to split the natural numbers into sets A and B such that for distinct elements $m, n \in A$ we have $m + n \in B$ and vice-versa.

(c) Given an invertible function f that satisfies

$$f^{-1}(1) = -2, \qquad f^{-1}(2) = 3,$$
$$f^{-1}(3) = 2, \qquad f^{-1}(4) = 5,$$
$$f^{-1}(5) = -7.$$

Solve for x:

i. [1 point] $f(x+2) = 5$.

ii. [1 point] $f(3x-4) = 3$.

iii. [1 point] $f(-5x) = 1$.

iv. [1 point] $f(-2-x) = 2$.

v. [1 point] $f(\frac{5}{x-1}) = 3$.

(d) Evaluate the following limits:

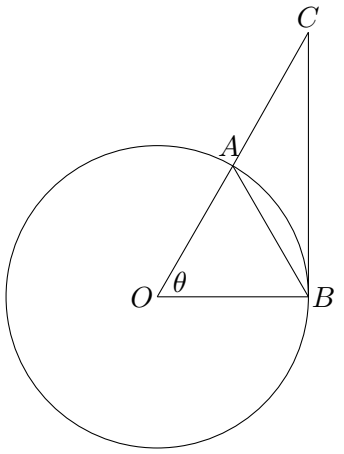
i. [5 points] $\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9}$.

ii. [5 points] $\lim_{x\to 0} \frac{\sin^2 x}{1-\cos x}$.

iii. [5 points] $\lim_{x\to \infty} \frac{2x^3+3x^2\cos x}{(x+2)^3}$.

Question 2 ..... *30 points*

(a) [10 points] Prove that $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$.



(b) [5 points] Find all real and complex solutions of the equation $z^4 + 2z^2 - 3 = 0$.

(c) [5 points] Find the cube roots of $-8i$.

(d) [5 points] If $f(x)$ has an x-intercept of $(3, 0)$ and a y-intercept of $(0, -5)$, what are the coordinates of x- and y-intercepts of $f^{-1}(x)$?

(e) [5 points] Use the $\epsilon - \delta$ definition to prove that $\lim_{x \to 2} x^2 = 4$.

Question 3 ..... *33 points*

(a) [6 points] What is the relative position of the following lines?

$$\begin{pmatrix}1\\3\\3\end{pmatrix} + t \cdot \begin{pmatrix}0\\1\\2\end{pmatrix} \text{ and } \begin{pmatrix}3\\2\\1\end{pmatrix} + t \cdot \begin{pmatrix}-1\\0\\1\end{pmatrix}$$

(b) [7 points] Does the following planes intersect? If so, what is the angle between them? Also, determine the parametric equation of the line which goes through them!

$$x + y + z = 1 \text{ and } x = y$$

(c) [6 points] Determine the ratio $AM : MF$ on Figure 1!

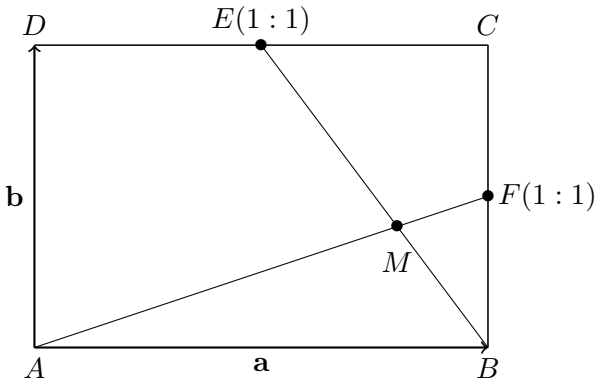


Figure 1:

(d) [4 points] Define the *cross product* of vectors!

(e) [5 points] Find the area of the Parallelogram with vertices $A(0,0), B(2,1), C(3,3)$ and $D(1,2)$.

(f) [5 points] What is the height of the former Parallelogram corresponding to the side A, B?

Question 4 ..... *22 points*

(a) [5 points] Let us suppose that $\lim_{n\to\infty} a_n = 0$ and $|b_n| \leq 2$. What is $\lim_{n\to\infty} a_n \cdot b_n = ?$

(b) Calculate the folllowing limits.

i. [3 points] $\lim_{n\to\infty} \frac{n}{n^2+1} = ?$

ii. [3 points] $\lim_{n\to\infty} \frac{n^2+2n+2}{1-n^2} = ?$

iii. [3 points] $\lim_{n\to\infty} \frac{\cos(n)^2+1}{n} = ?$

iv. [3 points] $\lim_{n\to\infty} \frac{\cos(n)+n+1}{n} = ?$

(c) [5 points] Is the following sequence bounded from above, from below, neither or both?

$$a_n = \frac{n^2-2}{n+1}$$