Math A1a - Calculus
Civil Engineering
2nd mid-term (Autumn 2016)

Name (Print): _____
Neptun Code: _____
Time Limit: 90 Minutes

This exam contains 14 pages (including this cover page) and 6 questions. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your Neptun code on the top of every page, in case the pages become separated.

At most **100** points can be collected in this exam. All questions have the same weight, there are no restrictions on your choice.

The following rules apply:

- You must **not** use your books, notes, or any other papers. You can **only** use blue or black pen, answers written with red pen, green pen or pencil will not be considered.
- You may use your calculator, but you are **not** allowed to use any other device, e.g. **your phone**.
- Organize your work, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- Mysterious or unsupported answers will not receive full credit. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- You have to work alone, any communications with the others will cause termination of your exam immediately.
- If you leave the room during the exam for any reason, then your exam will be terminated.
- If you need more space, ask the examiner to provide some blank papers.

Question	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
Total:	100	

Do not write in the table to the right.

With best wishes,
Dr. Mohamed Khaled and Gábor Borbély
November 24, 2016

Question 1 ..... *20 points*

(a) [5 points] State the Intermediate Value Theorem.

(b) [5 points] Find a positive integer n for which $x^5 + x^3 + 2x = 2x^4 + 3x^2 + 4$ has a solution somewhere in the interval $(n, n+1)$.

(c) [10 points] State and prove Bolzano's Theorem.

Question 2 ..... *20 points*

(a) [6 points] Does the graph of $y = x^4 - 2x^2 + 2$ have any horizontal tangents? If so, where? Does the graph of the same function have any vertical tangents? If so, where? Does it have vertical normals? Does it have horizontal normals?

(b) [6 points] Which rectangle of area 100 m^2 minimizes its height plus two times its length?

(c) [8 points] Sketch the graph of the function $y = \dfrac{1}{1+x^4}$.

Question 3 ..... *20 points*

(a) [5 points] State L'Hospital's rule.

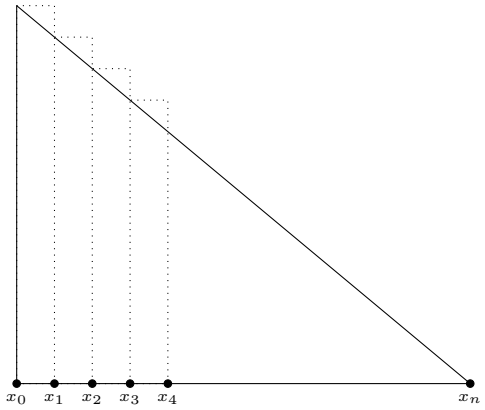
(b) [5 points] Prove L'Hospital rule for the "$\frac{0}{0}$" case at a finite limit point.

(c) [5 points] $\lim_{x \to 0} \frac{x^2 \tan(2x)}{\tan(3x)(\cos(x)-1)} = ?$

(d) [5 points] $\lim_{x \to -\infty} e^x \cdot \sin(x) \cdot x = ?$

Question 4 ..... *20 points*

(a) [10 points] Consider an isosceles right triangle such that both the twin sides are of length 1. One of the twin sides is placed horizontally such that the other twin meets its left endpoint. The horizontal side is divided into n-many equal segments. On each of these segments, a rectangle, of height equal to the distance between the left endpoint of segment and the hypotenuse, is placed. Let S_n be the sum of the areas of all of those rectangles. Show that $\lim_{n\to\infty} S_n$ is equal to the area of the triangle.



(b) [10 points] Use the definition of definite integral (Riemann Sum) to evaluate

$$\int_{-5}^{-1} (x^2 + 3x + 5)\,\mathrm{d}x.$$

(Specify which rule you will apply, you can use the sum formulas in the last page.)

Question 5 ..... *20 points*

(a) Evaluate the following definite integrals:

i. [$2\frac{1}{2}$ points] $\int_{-1}^{1} x^3 \, \mathrm{d}x$.

ii. [$2\frac{1}{2}$ points] $\int_{-1}^{1} | \, x \, | \, \mathrm{d}x$.

(b) [5 points] Evaluate the indefinite integral $\int \frac{x}{\sqrt{1-x}} \, \mathrm{d}x$.

(c) [5 points] Evaluate $\int 1 - \sin^2(x+1)\,\mathrm{d}x$.

(d) [5 points] Evaluate $\int \dfrac{5}{x^2+x-30}\,\mathrm{d}x$.

Question 6 ..... *20 points*

(a) [5 points] Prove that $\tanh(x)' = \frac{1}{\cosh^2 x}$ using the derivative of the quotient.

(b) [5 points] $\int \frac{e^x + 1}{e^{2x}} \, \mathrm{d}x = ?$

(c) [10 points] $\displaystyle\int \frac{x^3}{x^2 - 4x + 8}\,\mathrm{d}x = ?$

Sum Formulas

- $\sum_{k=1}^{n} c = nc$
- $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$
- $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$.
- $\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$.
- $\sum_{k=1}^{n} r^{k-1} = \sum_{k=0}^{n-1} r^k = \frac{1-r^n}{1-r}$ if $r \neq 1$.