Math A1a - Calculus
Civil Engineering
1st mid-term (Autumn 2016)

Name (Print): _____
Neptun Code: _____
Time Limit: 90 Minutes

This exam contains 13 pages (including this cover page) and 5 questions. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your Neptun code on the top of every page, in case the pages become separated.

You are required to show your work on each problem on this exam. The following rules apply:

- You must **not** use your books, notes, or any other papers. You may use your calculator but you are **not** allowed to use any other electronic devices.
- Organize your work, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- Mysterious or unsupported answers will not receive full credit. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.
- Use black or blue pen only.
- You have to work alone, any communications with the others will be revarded with zero credits.
- If one leaves the room within the given time frame then his/her work is considered finished.

Question	Points	Score
1	20	
2	20	
3	30	
4	32	
5	23	
Total:	100	

Do not write in the table to the right.

With best wishes,
Dr. Mohamed Khaled and Gábor Borbély
2016 October 19$^{\text{th}}$

Question 1 ..... *20 points*

(a) [3 points] Generate the truth table for the formulas $(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow p)$.

(b) [3 points] Verify the logical consequence: $p \vee (\neg q \wedge r) \models (q \vee \neg r) \rightarrow p$.

(c) Given an invertible function f that satisfies

$$f^{-1}(1) = -2, \qquad f^{-1}(2) = 3,$$
$$f^{-1}(3) = 2, \qquad f^{-1}(4) = 5.$$

Solve for x:

i. [1 point] $f(3x - 4) = 3$.

ii. [1 point] $f(-5x) = 1$.

iii. [1 point] $f(-2 - x) = 2$.

iv. [1 point] $f(\frac{5}{x-1}) = 3$.

(d) [5 points] Let f be the function defined by

$$y = f(x) \iff y \text{ is the largest solution of } y^2 = 3x^2 - 2xy.$$

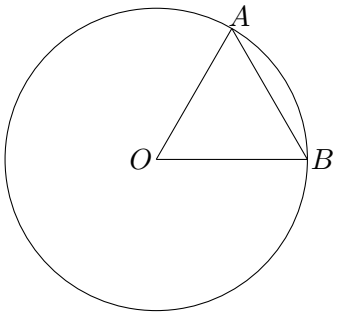
Find a formula (rule) for f. What are the domain and range of f?

(e) [5 points] Use the $\epsilon - \delta$ definition to prove that $\lim_{x \to 2} x^2 = 4$.

Question 2 ..... *20 points*

(a) [7 points] Prove that $\lim_{\theta \to 0} \sin\theta = 0$.

(Hint: The area of the triangle OAB is contained in the circular wedge OAB in the unit circle)



(b) [4 points] Use (a) to prove that $\lim_{\theta \to 0} \cos\theta = 1$.

(c) [4 points] Use (a) and (b) to prove that $\sin x$ is a continuous function.
(Hint: Use the fact $\sin(x+y) = \sin x \cos y + \cos x \sin y$.)

(d) [5 points] Let A, B and C be different sets containing letters of the alphabet. Explain why there must exist some letter that is either contained in exactly one of the sets or contained in exactly two of the sets.

Question 3 ..... *30 points*

(a) [3 points] Find all real and complex solutions of the equation $z^3 + 8 = 0$.

(b) [3 points] Find all real and complex solutions of the equation $z^4 + 2z^2 - 3 = 0$.

(c) [4 points] Find the cube roots of $-8i$.

(d) Find each of the following limits or show that it does not exist:

i. [$2\frac{1}{2}$ points] $\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9}$.

ii. [$2\frac{1}{2}$ points] $\lim_{x\to 0} \frac{\sin^2 x}{1-\cos x}$.

iii. [5 points] $\lim_{x\to 5} \frac{|x-5|}{x-5}$.

(e) Find each of the following limits or show that it does not exist:

i. [$2\frac{1}{2}$ points] $\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9}$.

ii. [$2\frac{1}{2}$ points] $\lim_{x\to 0} \frac{\sin^2 x}{1-\cos x}$.

iii. [5 points] $\lim_{x\to 5} \frac{|x-5|}{x-5}$.

Question 4 ..... *32 points*

(a) [5 points] Find the intersection of the planes $S : x + y + z = 1$ and $T : x = y$.

(b) [7 points] Does the following lines intersect? If so, what is the angle between them? Also, determine the equation of the plane which goes through them!

$$\begin{pmatrix}1\\2\\3\end{pmatrix} + t \cdot \begin{pmatrix}0\\1\\2\end{pmatrix} \text{ and } \begin{pmatrix}3\\2\\1\end{pmatrix} + t \cdot \begin{pmatrix}-1\\0\\1\end{pmatrix}$$

(c) [6 points] Determine the ratio $AM : MF$ on Figure 1!

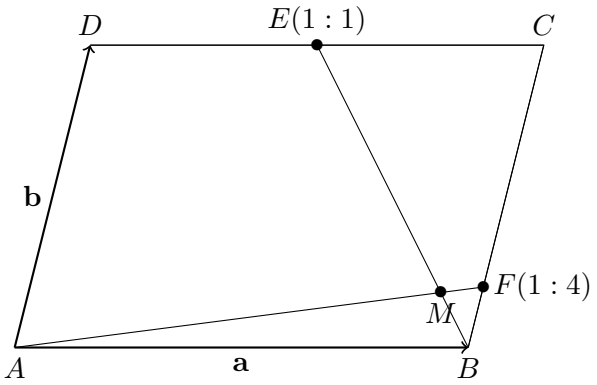
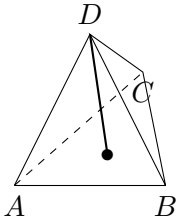


Figure 1:

(d) [4 points] Define the *triple product* of vectors!

(e) [5 points] Find the volume of the Tetrahedron with vertices $A(1,2,3), B(4,0,0), C(0,0,-1)$ and $D(1,1,1)$.

(f) [5 points] What is the height of the former Tetrahedron corresponding to the side A, B, C?



Question 5 ..... *23 points*

(a) [5 points] Let us suppose that $\lim_{n\to\infty} a_n = 0$ and $-1 \leq b_n \leq 1$. What is $\lim_{n\to\infty} a_n \cdot b_n =$?

(b) Calcualte the folllowing limits.

i. [3 points] $\lim_{n\to\infty} \frac{n}{n^2+1} = ?$

ii. [3 points] $\lim_{n\to\infty} \frac{n+1}{n+2} = ?$

iii. [3 points] $\lim_{n\to\infty} \frac{\cos(n)}{n} = ?$

iv. [4 points] $\lim_{n\to\infty} \frac{\sqrt{\cos(n)+n+1}}{n} = ?$

(c) [5 points] Is the following sequence bounded from above, from below, neither or both?

$$a_n = \frac{n^2+1}{n-1}$$