

Informatics 2, 3rd midterm (2020-05-19)

Submit the solutions to hazi@math.bme.hu, deadline: 19:15 CEST

1. Add a new method to the `Tree` class that represents binary trees. The new method should be called `edges` and return the number of edges in the tree. (5 points)

Example:

```
t = Tree(3)
for i in [1, 2, 0, -1, 4, 6, 5]:
    t.insert(i)
print(t.edges())
```

Result: 7

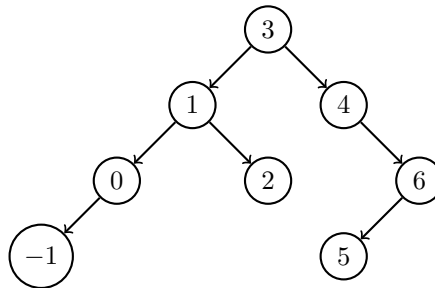


Figure 1: The following edges were inserted (in order): [3,1,2,0,-1,4,6,5]

Hint: 11th Lab.

2. Let's look at the following triangle shaped grid:

1			
1	1		
1	2	2	
1	3	5	5

Figure 2: How many ways can you go from the top-left corner to each cell if you can step only down and right?

Write a function that calculates how many different paths lead to a given cell from the topmost cell if you can move only downward and to the right. You can see the answer inside each cell in this triangle. (5 points)

- The function should be called `steps`.
- With one parameter, an integer: the size of the triangle.
- The return value should be a list of lists, where every element contains the answer to that particular cell. In the example:

`steps(4) ↦ [[1], [1, 1], [1, 2, 2], [1, 3, 5, 5]]`

Hint: similarly to binomial coefficients, every cell is the sum of to cell above it and to the left of it, except on the sides of the triangle.

3.

- Write a function that replaces parenthesis in a string to the `$` character, but there will be two kind parenthesis: `()` and `{}`. You have to replace everything inside any top-level parenthesis regardless of the type. (5 points)
- The function should be called `parenthesis2`.
- It should have one parameter: the input string with both type of parenthesis.
- Return a string with the same length, where the parentheses are replaced.

Example:

`parenthesis2("1/(2+(-3))-{4+(-5)}") ↦ "1/$$$$$$$-$$$$$$$$"`

4. Write a function that generates random 3 dimensional unit vectors, use `numpy`.

(5 points)

- the function should be called `unit3`.
- With one parameter: n the number of vectors we want.
- Return a $\mathbb{R}^{n \times 3}$ array where each row is a 3 dimensional unit vector.

Example:

```
>>> unit3(2)
array([[ -0.74790227, -0.47255394,  0.46619199],
       [ 0.19888237, -0.69017507,  0.69577595]])
```

Hint: generate an $n \times 3$ matrix with values in $[-1, 1]$ then divide each row by its norm.