Informatics 1
Lecture 2: Abstract machines

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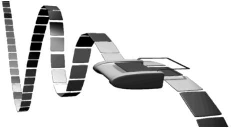
- A Turing machine can be defined by $M = \langle Q, \Gamma, b, \Sigma, \delta, q_0, F \rangle$, where
- Q is the non-empty set of "states",
- Γ the finite, non-empty "tape alphabet",
- $b \in \Gamma$ the "blank symbol" (the only symbol allowed to occur on the tape infinitely often),
- $\Sigma \subseteq \Gamma \setminus \{b\}$ the set of "input symbols",
- $q_0 \in Q$ the "initial state"
- $F \subseteq Q$ the set of "final states" (this is when the machine stops),
- $\delta : (Q \setminus F) \times \Gamma \hookrightarrow Q \times \Gamma \times \{L, R\}$ is a partial function called the "transition function", where L is left shift, R is right shift (moves the tape)

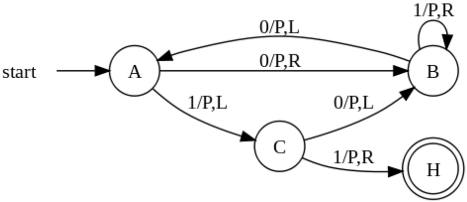


Turing completeness

- *Church–Turing-thesis* (30's) every formalizable problem that can be solved by any means with some algorithm, can be solved with a Turing machine.
- A computational/data manipulation system is Turing complete if it can implement any Turing machine.

- Busy beaver (Tibor Radó, 1962) A Turing machine with some constraints that writes the most non-empty symbols on an empty tape, and halts in finite steps.
- $Q = \{A, B, C, HALT\}$
- $\Gamma = \{0, 1\}$
- $b = 0$ (empty symbol)
- $\Sigma = \{1\}$
- $q_0 = A$ (initial state)
- $F = \{HALT\}$
- δ table:

	A	B	C
0	1RB	1LA	1LB
1	1LC	1RB	1RH



1	A	0 0 0 0 0	0	0 0 0 0 0 0
2	B	0 0 0 0 0	0	1 0 0 0 0 0
3	A	0 0 0 0 1	1	0 0 0 0 0 0
4	C	0 0 0 1 1	0	0 0 0 0 0 0
5	B	0 0 1 1 1	0	0 0 0 0 0 0
6	A	0 1 1 1 1	0	0 0 0 0 0 0
7	B	0 0 1 1 1	1	1 0 0 0 0 0
8	B	0 0 0 1 1	1	1 1 0 0 0 0
9	B	0 0 0 0 1	1	1 1 1 0 0 0
10	B	0 0 0 0 0	1	1 1 1 1 0 0
11	B	0 0 0 0 0	0	1 1 1 1 1 0
12	A	0 0 0 0 1	1	1 1 1 1 0 0
13	C	0 0 0 1 1	1	1 1 1 0 0 0
14	H	0 0 0 1 1	1	1 1 1 0 0 0

- The RAM-machine consists of a p program register and an r data register, both of them indexed by natural numbers, the data register contains zeros initially.
- The execution of the program starts with executing the command in cell p_0 and ends with an empty command.
- The contents of the ith cell of the data register ($i \in \mathbb{N}_0$) is denoted by $r[i]$ or r_i, these can only contain integers.
- These are the possible commands, where $z \in \mathbb{Z}$, $i, n \in \mathbb{N}_0$:

  $r_i \leftarrow z$

  $r_i \leftarrow r_n$, $r_i \leftarrow r_{r_n}$ (same as $r_i \leftarrow r[r[n]]$),

  $r_i \leftarrow r_i \pm r_n$, ($r_i \leftarrow r_i * r_n$, $r_i \leftarrow r_i / r_n$),

  p_n: jump to the nth program line,

  if $r_i = 0$ p_n: jump to the nth program line if $r_i = 0$,

  if $r_i > 0$ p_n: jump to the nth program line if $r_i > 0$, For this lecture let us use this "computer like" RAM-machine:

- The program register and memory is finite,
- every memory cell is 1 byte long, every program line is 2 bytes long, the first byte contains the command and the second byte contains the operand, i.e.

  `ADD 12` means: $r_0 \leftarrow r_0 + r_{12}$
- every calculation is done with the 0th memory cell (and sometimes another one),
- we use mnemonikokkal for the commands, there are three types:
  - explicit: the operand n is a number (denoted by an $=$ at the end of the expression)
  - direct: the operand n is a memory cell, the operation is done with the contents of $r[n]$,
  - indirect: the operand n is the index of a memory cell, the operation is done with $r[r[n]]$ (denoted by a * at the end of the expression)

Controller commands		
JUMP	n	jump to the nth command
JZERO	n	jump to the nth command if $r_0 = 0$
JGTZ	n	jump to the nth command if $r_0 > 0$
HALT		stop

Arithmetic commands

direct			*indirect*			*explicit op*		
ADD	n	$r_0 \leftarrow r_0 + r_n$	ADD*	n	$r_0 \leftarrow r_0 + r_{r_n}$	ADD=	n	$r_0 \leftarrow r_0 + n$
SUB	n	$r_0 \leftarrow r_0 - r_n$	SUB*	n	$r_0 \leftarrow r_0 - r_{r_n}$	SUB=	n	$r_0 \leftarrow r_0 - n$
MULT	n	$r_0 \leftarrow r_0 * r_n$	MULT*	n	$r_0 \leftarrow r_0 * r_{r_n}$	MULT=	n	$r_0 \leftarrow r_0 * n$
DIV	n	$r_0 \leftarrow r_0 / r_n$	DIV*	n	$r_0 \leftarrow r_0 / r_{r_n}$	DIV=	n	$r_0 \leftarrow r_0 / n$

Data manipulation, IO

direct			*indirect*			*explicit op*		
LOAD	n	$r_0 \leftarrow r_n$	LOAD*	n	$r_0 \leftarrow r_{r_n}$	LOAD=	n	$r_0 \leftarrow n$
STORE	n	$r_n \leftarrow r_0$	STORE*	n	$r_{r_n} \leftarrow r_0$			
READ	n	reads n numbers from the input into $r_1, r_2, \ldots, r_n$						
WRITE	n	writes n numbers to the output from $r_1, r_2, \ldots, r_n$						

Write a program to calculate (a, b) (greatest common divisor), where $a, b \in \mathbb{N}_0$!

p	command	operand	notes
0	LOAD =	12	
1	STORE	1	r[1] <- a
2	LOAD =	16	
3	STORE	2	r[2] <- b
4	JZERO	17	
5	LOAD	1	r[0] <- r[1]
6	DIV	2	r[0] <- ⌊a/b⌋
7	STORE	3	r[3] <- ⌊a/b⌋
8	MULT	2	
9	STORE	4	r[4] <- b*⌊a/b⌋
10	LOAD	1	
11	SUB	4	r[0] <- a - b*⌊a/b⌋ = a mod b
12	STORE	5	
13	LOAD	2	
14	STORE	1	r[1] <- b
15	LOAD	5	b <- a mod b
16	JUMP	3	
17	LOAD	1	
18	STORE	6	this is (a,b)
19	HALT	0	

What about real machines?

- Machine code: binary sequence that the processor can directly interpret.
- Every code written in any programming language compiles into machine code.
- The closest language to machine code is assembly, it is a bit similar to what we described as an example to the RAM-machine. The compiler for assembly is called assembler.
- Registers are special CPU memories with extremely fast read-write speeds, but very limited capacity.

Questions

1. What are the basics of a Turing-machine?
2. What does the "busy beaver" do?
3. What's the difference between the direct and the indirect commands?
4. What is the machine code, assembly and the assembler?
5. What are the contents of the memory after issuing these commands?

```
1   LOAD=    5
2   STORE    1
3   STORE*   1
4   JZERO    7
5   LOAD=    2
6   MUL      1
7   HALT
```