Informatics 1
Lecture 2: Abstract machines

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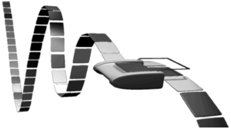
1. Turing machine

2. RAM-machine (random access machine)

- A Turing machine can be defined by $M = \langle Q, \Gamma, b, \Sigma, \delta, q_0, F \rangle$, where



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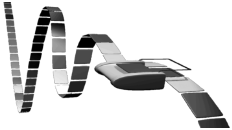

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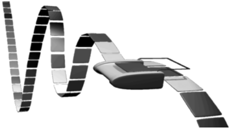

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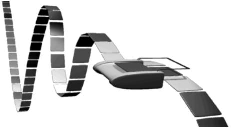

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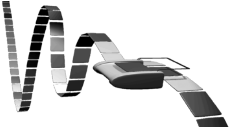

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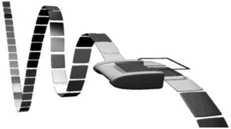

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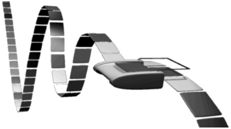

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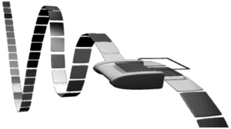

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- $\delta : (Q \setminus F) \times \Gamma \hookrightarrow Q \times \Gamma \times \{L, R\}$ is a partial function called the "transition function", where L is left shift, R is right shift (moves the tape)

Turing completeness

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- A computational/data manipulation system is Turing complete if it can implement any Turing machine.

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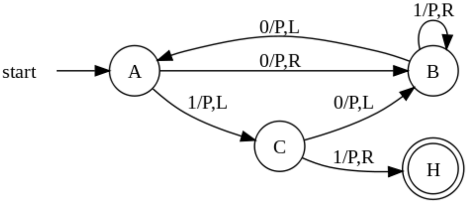

1	A	0 0 0 0 0	0	0 0 0 0 0 0
2	B	0 0 0 0 0	0	1 0 0 0 0 0
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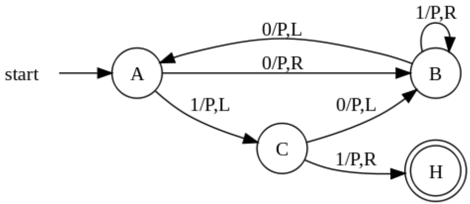

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4	C	0 0 0 1 1 |0| 0 0 0 0 0 0
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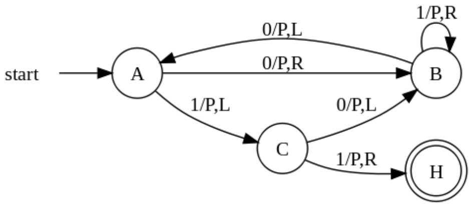

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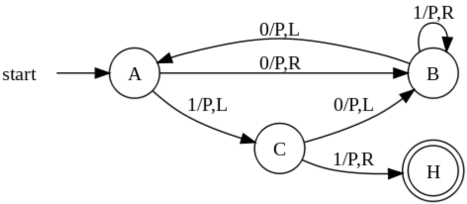

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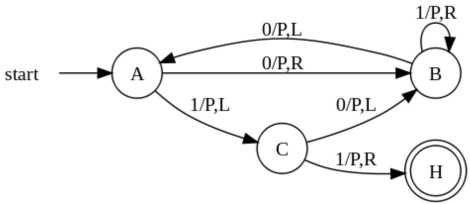

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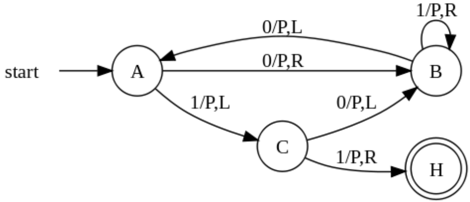

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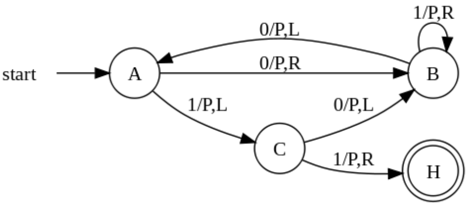

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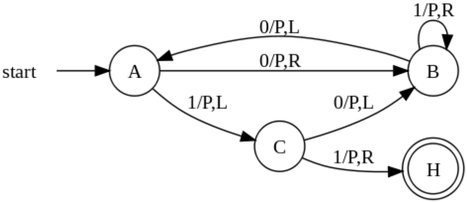
	A	B	C
0	1RB	1LA	1LB
1	1LC	1RB	1RH



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For this lecture let us use this "computer like" RAM-machine:

- The program register and memory is finite,
- every memory cell is 1 byte long, every program line is 2 bytes long, the first byte contains the command and the second byte contains the operand, i.e.

  `ADD 12` means: $r_0 \leftarrow r_0 + r_{12}$
- every calculation is done with the 0th memory cell (and sometimes another one),
- we use mnemonikokkal for the commands, there are three types:
  - explicit: the operand n is a number (denoted by an $=$ at the end of the expression)
  - direct: the operand n is a memory cell, the operation is done with the contents of $r[n]$,
  - indirect: the operand n is the index of a memory cell, the operation is done with $r[r[n]]$ (denoted by a * at the end of the expression)

Controller commands

JUMP	n	jump to the nth command
JZERO	n	jump to the nth command if $r_0 = 0$
JGTZ	n	jump to the nth command if $r_0 > 0$
HALT		stop

Arithmetic commands

direct			*indirect*			*explicit op*		
ADD	n	$r_0 \leftarrow r_0 + r_n$	ADD*	n	$r_0 \leftarrow r_0 + r_{r_n}$	ADD=	n	$r_0 \leftarrow r_0 + n$
SUB	n	$r_0 \leftarrow r_0 - r_n$	SUB*	n	$r_0 \leftarrow r_0 - r_{r_n}$	SUB=	n	$r_0 \leftarrow r_0 - n$
MULT	n	$r_0 \leftarrow r_0 * r_n$	MULT*	n	$r_0 \leftarrow r_0 * r_{r_n}$	MULT=	n	$r_0 \leftarrow r_0 * n$
DIV	n	$r_0 \leftarrow r_0 / r_n$	DIV*	n	$r_0 \leftarrow r_0 / r_{r_n}$	DIV=	n	$r_0 \leftarrow r_0 / n$

Data manipulation, IO

direct			*indirect*			*explicit op*		
LOAD	n	$r_0 \leftarrow r_n$	LOAD*	n	$r_0 \leftarrow r_{r_n}$	LOAD=	n	$r_0 \leftarrow n$
STORE	n	$r_n \leftarrow r_0$	STORE*	n	$r_{r_n} \leftarrow r_0$			

READ	n	reads n numbers from the input into $r_1, r_2, \ldots, r_n$
WRITE	n	writes n numbers to the output from $r_1, r_2, \ldots, r_n$

Write a program to calculate (a, b) (greatest common divisor), where $a, b \in \mathbb{N}_0$!

p	command	operand	notes
0	LOAD =	12	
1	STORE	1	r[1] <- a
2	LOAD =	16	
3	STORE	2	r[2] <- b
4	JZERO	17	
5	LOAD	1	r[0] <- r[1]
6	DIV	2	r[0] <- ⌊a/b⌋
7	STORE	3	r[3] <- ⌊a/b⌋
8	MULT	2	
9	STORE	4	r[4] <- b*⌊a/b⌋
10	LOAD	1	
11	SUB	4	r[0] <- a - b*⌊a/b⌋ = a mod b
12	STORE	5	
13	LOAD	2	
14	STORE	1	r[1] <- b
15	LOAD	5	b <- a mod b
16	JUMP	3	
17	LOAD	1	
18	STORE	6	this is (a,b)
19	HALT	0	

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- The closest language to machine code is assembly, it is a bit similar to what we described as an example to the RAM-machine. The compiler for assembly is called assembler.
- Registers are special CPU memories with extremely fast read-write speeds, but very limited capacity.

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3. What's the difference between the direct and the indirect commands?
4. What is the machine code, assembly and the assembler?
5. What are the contents of the memory after issuing these commands?

```
1   LOAD=    5
2   STORE    1
3   STORE*   1
4   JZERO    7
5   LOAD=    2
6   MUL      1
7   HALT
```