

Let $f(x) = \sqrt{1-x^2}$. The formula for curvature is.

$$\kappa(x) = \frac{f''(x)}{\left(1 + f'(x)^2\right)^{\frac{3}{2}}}$$

First calculate f' .

$$f'(x) = \frac{1}{2 \cdot \sqrt{1-x^2}} \cdot (-2x) = \frac{-x}{\sqrt{1-x^2}}$$

Then calculate f'' .

$$f''(x) = \frac{-\sqrt{1-x^2} - (-x) \cdot \frac{1}{\sqrt{1-x^2}} \cdot (-2x)}{1-x^2} = \frac{-\sqrt{1-x^2} - x^2 \frac{1}{\sqrt{1-x^2}}}{1-x^2}$$

With these:

$$\begin{aligned} \frac{f''(x)}{\left(1 + f'(x)^2\right)^{\frac{3}{2}}} &= \frac{\frac{-\sqrt{1-x^2} - x^2 \frac{1}{\sqrt{1-x^2}}}{1-x^2}}{\left(1 + \left(\frac{-x}{\sqrt{1-x^2}}\right)^2\right)^{\frac{3}{2}}} = \frac{\frac{-\sqrt{1-x^2} - x^2 \frac{1}{\sqrt{1-x^2}}}{1-x^2}}{\left(1 + \frac{x^2}{1-x^2}\right)^{\frac{3}{2}}} = \frac{\frac{-\sqrt{1-x^2} - x^2 \frac{1}{\sqrt{1-x^2}}}{1-x^2}}{\left(\frac{1}{1-x^2}\right)^{\frac{3}{2}}} = \\ &= (1-x^2)^{\frac{3}{2}} \cdot \frac{-\sqrt{1-x^2} - x^2 \frac{1}{\sqrt{1-x^2}}}{1-x^2} = (1-x^2)^{\frac{1}{2}} \cdot \left(-\sqrt{1-x^2} - x^2 \frac{1}{\sqrt{1-x^2}}\right) = \\ &= -(1-x^2) - x^2 = -1 \end{aligned}$$