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COGNITIVE DEVELOPMENT

The construction of number concepts

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Abstract

This paper describes the development of number concepts from infancy to early childhood. The results of a diary study on a child's one-to-one correspondence activities from 12 to 38 months of age are presented. The diary study suggested that social activities, such as distributing objects to people, play a greater role in early numerical development than conservation-like activities, such as matching object sets. There also was evidence that early number concepts are highly context-dependent. Specifically, although this child represented and matched equivalent sets in a few highly constrained contexts, he could not do so in others. An alternative to the competence–performance distinction is developed for explaining such cross-task variability.

Keywords: One-to-one correspondence; Number concepts; Variability

1. Introduction

It has been 60 years since Piaget published his landmark book, "The Child's Conception of Number." In that time, researchers have debated the competence of infants, the meaning of number conservation, and the reasons that children demonstrate numerical competence in some situations but not others. But despite deeply opposed stands on these issues, most researchers agree that number concepts are at least partly constructed from environmental input. Even those who have argued that numerical development is promoted by an innate counting ability do not claim that number concepts emerge full-blown. For example, Gelman (1998) wrote, "As our account is fundamentally committed to the premise that concept learning is what happens as a function of experience, it is a learning account . . . it

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takes as given that learners must encounter domain-relevant experiences, even for core domains. Without the opportunity to interact with and store relevant data, there cannot be a forward moving construction of the knowledge of a domain, be it core or noncore in kind" (pp. 564–565).

Given the general agreement on this point, surprisingly little progress has been made in explaining *how* number concepts are constructed. Although the literature has produced thorough investigations of children's performance on the number conservation task, as well as the development of conventional counting, few studies have focused on what happens between infancy and early childhood to support these developmental changes. If number concepts are constructed, what are the processes and experiences that allow this construction to occur?

The present paper is aimed at addressing this question. I will present the results of a diary study that tracked the one-to-one correspondence activities of a boy from 12 to 38 months of age. This study not only indicates the range and relative frequency of various one-to-one experiences, but also reveals that initial number concepts are highly context-dependent. That is, numerical competence can appear full-blown in one context but nonexistent in another. I develop a conceptualization of this cross-task variability that does not require a separation of competence from performance. I also identify specific processes that may promote the abstraction of number concepts across different contexts. The goal is to clarify what counts as relevant input for number concepts and the mechanisms by which this information is transformed into coherent conceptual structures.

2. Quantification in infancy: initial assumptions

The origins of numerical development have received considerable attention in the literature. In fact, one of the most contentious issues in recent years has been how much numerical competence should be attributed to infants. Because this issue is discussed at length elsewhere (Mix, Huttenlocher, & Levine, 2002a, 2002b), I will not provide a comprehensive review here. However, before I proceed to a discussion of quantification in toddlerhood and early childhood, I would like to make explicit some initial assumptions about developmental origins.

First, I assume that newborn infants are equipped to learn about number with nothing more than the same basic sensory, motor, and learning capacities they bring to any other task. This is the same starting point adopted in classic views of numerical development — that is, the view that number concepts are learned like any other concept, through a gradual induction over experience (e.g., Bruner et al., 1966; Piaget, 1941/1965). An influential alternative has been that numerical development is guided by innate knowledge structures (e.g., Gelman, 1991; Wynn, 1995). However, a recent review of the literature revealed no clear evidence that infants are sensitive to discrete number (Mix et al., 2002a). Instead, infants' performance on quantitative tasks can be explained in terms of other quantitative cues, such as contour length. Thus, there is to date no compelling reason to build more than basic perceptual and learning abilities into the infant.

However, there is reason to believe that an awareness of discrete objects has developed by the end of the infancy period. Although this does not mean older infants represent specific set sizes, studies indicate that infants gradually learn to tell one object from another based on various cues over the period of 4–12 months of age (Wilcox, 1999; Xu & Carey, 1996). Object individuation implies a sense of discreteness that could form the foundation for subsequent number concepts. In fact, this would be consistent with Piaget's original claim that number concepts are built upon object concepts. The main point for now is that the ability to perceive sets as discrete objects rather than (or in addition to) perceiving them in terms of non-numerical cues is likely present by the end of infancy. In the remainder of this paper, I consider how children might pull together concepts of numerosity from such beginnings.

3. Grist for the mill: how do children experience one-to-one correspondence?

If children induce number concepts from direct experience, what experiences are relevant? Those involving one-to-one correspondence are undoubtedly among the most informative. One-to-one correspondence is a defining aspect of number and numerical relations, such as equivalence and ordinality. It is implicated in all current models of nonverbal quantification including the mental models view (Huttenlocher, Jordan, & Levine, 1994), object tokens models (Simon, 1997; Uller, Carey, Huntley-Fenner, & Klatt, 1999), and preverbal counting (Gallistel & Gelman, 1992). An understanding of one-to-one correspondence is needed to count correctly (Gelman & Gallistel, 1978) and to conserve number (Piaget, 1941/1965).

Previous studies have focused on whether children use one-to-one correspondence to make logical inferences about numerical invariance and equivalence. For example, Russac (1978) asked children to construct a set of chips that was numerically equivalent to a set of dots on a stimulus card. To ensure that they used one-to-one correspondence, children were asked not to count or place the chips directly on top of the dots. Russac found that children performed very poorly on this task until second grade. In fact, children in his study performed worse on this task than they did on Piaget's number conservation task. Clearly, it takes some time for children to appreciate the logical implications of one-to-one correspondence in these situations. But, how do they build up to that understanding? What naturalistic experiences provide grist for the conceptual mill? To address this question, I carried out a 26-month longitudinal diary study of the one-to-one correspondence activities performed by my son, Spencer.

Data collection began in January 2000, when Spencer was 12 months old and continued until March 2002, when he was 38 months old. He was an only child throughout this period. Until 18 months of age, he received parental care exclusively. Between 18 and 38 months of age, an in-home nanny supervised him for 12–15 h a week. Data were recorded in three daily planners — one each for the years 2000, 2001, and 2002. All spontaneous one-to-one correspondence activities were noted using either a handheld tape recorder or notepad. These notes were later transcribed into the appropriate planner according to the observation date.

3.1. The range of possibilities

The initial focus of the diary was on object correspondences — the kind tested in previous research (e.g., Piaget's number conservation task). Thus, I watched with particular vigilance for Spencer's spontaneous experiences with aligned, or paired, sets of objects. However, my focus broadened throughout the first few months as I noticed the variety of one-to-one activities Spencer regularly performed. I ultimately identified six categories. These were based on the activities that he initiated himself and not from direct instruction or examples of one-to-one correspondence that he might have observed. For example, if I set the table in front of him, I did not count this as a one-to-one correspondence activity even though he may well have learned from such input. However, if he spontaneously passed out silverware, this was recorded. I drew this distinction because, whereas Spencer's attention was surely engaged by his self-generated activities, it was more difficult to gauge the extent to which this held for externally generated inputs. Also, to maintain natural interactions with Spencer and focus my attention fully on his behavior, it seemed preferable not to record and analyze my own activities.

I further restricted my observations to situations that involved sets of at least two items. In the broadest sense, any pairing could be considered a one-to-one correspondence. For example, every time a child names an object with the correct word, this could be considered an instance of one-to-one correspondence. Indeed, such experiences may play an important role in numerical development. However, for the present purposes, I restricted my observations to correspondences between sets containing multiple items. My reasoning was that because such experiences involved different set sizes, they would be more relevant for developing notions of cardinal number, equivalence, and ordinality than simple one-to-one pairings.

The six categories are presented below starting with the object correspondences usually tested in experimental tasks and moving gradually toward correspondences without objects. Table 1 provides examples of each activity drawn from Spencer's diary.

1. *Aligned objects*: The spatial alignment of individual items from two different sets of objects. This could take the form of matched rows, as in Piaget's number conservation task, nonlinear configurations consisting of pairs from two sets, or individual items from different sets placed in physical contact.
2. *Objects with slots*: Alignments between a set of free-moving objects and a corresponding set of holes or slots, such as eggs in a carton.

Table 1
Examples of one-to-one correspondence activities

Aligned objects

8/20/00 (19 months): At Macri's for lunch, I ordered Spencer a chocolate milk and a water. The waiter brought the drinks in two identical small cups with lids and straws — setting each on a cocktail napkin. After drinking for several minutes, Spencer put the water cup carefully back on its napkin. Then, he tried to put the milk back on its napkin, but it was out of his reach. When I slid it back, he carefully put the cup on it and nodded.

7/15/01 (30 months): When I came into the bedroom I saw that Spencer had placed one of my shoes squarely on top of one of Brian's slippers and, two feet away, had placed the other shoe in that pair squarely on top of Brian's other slipper.

Objects with slots

11/8/00 (22 months): Spencer took a handful of graham crackers and placed one each in a four-compartment tray. He held the extras in his hand.

8/8/01(30 months): At a restaurant, the waitress brought out a wooden shoe that held four crayons upright in four circular holes. Spencer repeatedly took all four crayons out and returned them one by one to the four holes.

Distributed objects

3/24/00 (14 months): Today Spencer gave a piece of cereal to Rocky and then got one for Mushu.

12/12/00 (23 months): At the breakfast table, Spencer handed out the silverware while saying, "Mommy spoon, Daddy spoon, Mommy fork, Daddy fork." He repeated this on several meals during our trip to California.

Tagging objects and people

9/30/00 (20 months): We were playing with play dough. I cut one long piece into about seven squares. Spencer made an indentation in each one with his finger. Then he touched each indentation.

12/12/00 (23 months): Going down the stairs, Spencer said, "step" on each step.

Tagging events

6/20/00 (17 months): Brian and Spencer were playing in the sandbox. Brian repeatedly held up a handful of sand and said, "3-2-1" before pouring it out as Spencer watched. A little later, Brian saw Spencer hold up a handful of sand and say, "uh-uh-uh" and then drop it on the third grunt.

4/6/01(26 months): I sneezed twice in a row. Spencer said, "Bless you. Bless you" and smiled.

Turn-taking

9/1/00 (19 months): I grabbed Spencer's foot and blew a raspberry on it. He gave me his other foot and I did it again. We went back and forth several times. Every time I did one foot once, he'd give me the other.

1/17/01(24 months): Spencer had two tupperware lids. He said, "Red! Orange!" — correctly labeling the two colors. Then, he put one behind his back and said, "Can't find red!" Then he brought the red one back around and said, "There it is." He repeated this several times, alternating between the red and the orange lid.

8/7/01 (30 months): Spencer and Brian were taking turns sitting in a beach chair. Spencer commented, "Daddy's turn" when Brian sat down, then "My turn" when he sat down, etc. for several repetitions.

3. *Distributed objects*: A one-to-one distribution of objects to people, animals, or dolls, as in handing out pieces of cake at a birthday party. Comments on existing distributions were included (e.g., Mommy's cup. Daddy's cup. Baby's cup.)
4. *Tagged objects and people*: Acting on each object or person in a group once and only once. The action could be nonverbal, as in the case of tickling each person in the room, or verbal, as in the case of counting objects or naming them in sequence.
5. *Tagged events*: Performing an action for or verbally tagging each event in a series of events. This was a very rare occurrence in Spencer's activities. The two examples noted in Table 1 are the only entries that reflected this activity.
6. *Turn-taking/alternating actions*: Performing actions in alternation with another person's actions, such as taking turns with a toy, or one's own, such as touching one toy and then another in a repeating sequence.

It is clear from the range of one-to-one correspondence activities described above that children have many sources of relevant input besides aligning objects. Children experience one-to-one correspondence in a variety of contexts some that do not involve objects at all. In fact, one striking aspect of these observations is the variety of ways children gain information about one-to-one correspondence in interactions with groups of people rather than through object manipulations alone.

3.2. *Order of appearance*

Table 2 presents the six categories in the order they appeared in Spencer's diary. The age of first entry is provided; however, the rank ordering is based on an average of the age in months for which the first two entries of each type were recorded. This was done to ensure that the rankings were not unduly affected by one exceptionally early entry.

The rankings indicate that Spencer's earliest one-to-one correspondence experiences involved distributions of objects. These were noted several months before any of the other five categories. Distributed objects are interesting because they result in a lasting correspondence that can be reviewed pair by pair, and not all at

Table 2
Age of onset for six one-to-one activities

Rank	Activity type	Age in months of first entry	Age in months averaged over first two entries
1	Distributed objects	13	16
2	Tagged objects	20	20
3	Turn-taking	19	20.5
4	Tagged events	17	21.5
5	Aligned objects	19	23.5
6	Objects with slots	22	25

once as two rows of aligned objects could be. For example, when Spencer passed out toys to all of the children playing in the living room, he could not easily see that the sets were symmetrical or equal in length or density. His evaluations of equivalence were reduced to local correspondences (e.g., Does Nicholas have a truck? Does Benjamin have a truck?). Another aspect of this activity is that it is highly social — certainly more so than pairing objects together. Thus, at least for Spencer, the earliest one-to-one activities appeared to be embedded in social interaction. In contrast, the last activities to appear were the ones that also have received the greatest attention in the literature — aligning objects and pairing objects with slots. These were both preceded by tagging objects, turn-taking, and tagging events — three activities that involve repeating actions rather than pairing objects with other objects or with recipients. This pattern suggests that object correspondences may be one of the last sources of numerical information children access.

3.3. *Relative frequency*

A similar pattern was obtained when the relative frequency of the six activities was examined. Frequency was estimated by tallying the number of diary entries for each activity type (see Table 3). There are two caveats to bear in mind when considering these data. First, as noted above, some categories were recognized as relevant after others, and thus, may have been underestimated. However, the categories most likely to have been underestimated were object distributions, turn-taking, and tagging, and not aligned objects or objects with slots. Thus, whereas the observed pattern of relative frequency would have been more pronounced had these categories been recognized sooner, the overall pattern probably would not have changed. Second, because the author and child were not in constant contact, all of the tallies are lower than the actual number of one-to-one activities Spencer performed. Therefore, although the relative frequencies are valid, the absolute numbers of entries should not be given too much weight.

As shown in Table 3, object distributions and tagging objects were far and away the most frequent activities. Thus, object distributions were not only the first to appear, but also among the most frequent. In fact, they may have been the single most frequent depending on how one defines “tagging objects.” The

Table 3
Frequency of six one-to-one activities

Rank	Activity type	Number of diary entries
1	Distributed objects	87
2	Tagged objects	73
3	Turn-taking	25
4	Objects with slots	22
5	Aligned objects	9
6	Tagged events	2

tally presented in Table 3 includes not only instances when Spencer acted upon objects one-to-one (see Table 1 for examples), but also his attempts to tag objects with individual count words. These attempts were considered "tagging objects" as long as he separated the count words and made a reasonable effort to match them with individual objects (i.e., a counting act was tallied even if he double counted or skipped a few of the items). However, reciting the count words without reference to objects, waving at a set while saying count words in rapid succession, and extremely errorful counting were not included in this category. If counting is excluded from this category altogether, the frequency of tagging activities drops to 34.

Another interesting finding is that the frequency of aligned objects was extremely low compared to most other categories. Recall that object alignment has had a central role in research on numerical equivalence and one-to-one correspondence. In light of that, it is surprising that Spencer spontaneously experienced very few of these correspondences. Instead, most of his information about one-to-one correspondence seemed to come from repeated actions or object distributions. This suggests that one reason young children fail Piaget's number conservation task is that their developing notions of one-to-one correspondence are embodied in completely different contexts.

3.4. Why are object alignments less accessible?

If object alignments are not a primary source of one-to-one correspondence information for young children, why is this the case? One possibility is that relations between object sets are more symbolic and arbitrary than the socially-mediated relations that make up the other categories. That is, there may be fewer natural relations between object sets that invite one-to-one pairings. Aligning such sets could be motivated by an interest in symmetry or pattern completion, but these relations may be less salient to young children than the social interactions involved in passing items out to people or taking turns.

Consistent with this interpretation, Spencer was much more likely to match objects to slots than he was to match objects to other objects. For example, he put all the pieces back in a knob puzzle more often than he lined all of his cars up with all of his trucks. In a sense, these object-to-slot matches are a perfect bridge between object distributions and object alignments because a hole in a puzzle board is rather like a person without a toy — it is a signal that something is needed. At the same time, like object alignments, matching objects to slots involves relating inanimate things. Therefore, performing such pairings does not have the social reinforcement that giving a toy or food to someone might.

An alternative reason that object alignments emerge relatively late is that local pairings may be easier to perform than aligning whole sets. When children distribute objects or tag individual items, they can focus on the one pairing at a time. In fact, it would be difficult to compare sets wholistically within these contexts because the sets are not simultaneously available to inspection. Either they are distributed over some distance and perhaps remain in motion (as in the case of toys distributed to playmates) or one of the sets is temporally distributed and intangible (as in the case of tagging objects with count words). In contrast, when children align two sets of objects, the overall correspondence between sets can be evaluated because both sets are available for inspection. However, whenever two sets are inspectible and within close proximity, children can compare them in a variety of ways besides one-to-one correspondence. Indeed, Piaget attributed preschoolers' errors in the number conservation task to an emphasis on global cues, such as line length. This suggests that, at least initially, children may have better access to the one-to-one relations embodied in local pairings that obscure the global relations between sets.

Although children may gain access to one-to-one correspondence through local pairings, such experiences probably yield little information about numerosity, numerical equivalence, or ordinality because the sets cannot be inspected and compared. Once again, the object-to-slot matches may act as a bridge, this time between local correspondences that are easier to perform and global correspondences that provide information about numerosity and equivalence. They are similar to object distributions and tagging in that they invite local pairings. However, like object alignments, the resulting distribution is lasting and available to inspection.

In fact, there is evidence from Spencer's diary that object-to-slot matches highlighted equality and inequality more than object distributions and tagging. When Spencer distributed objects, he rarely commented on numerical equivalence. He said, "There you go," or "That's Mommy's" but did not often comment on the need for additional objects to reach equivalence. In fact, it was not unusual for him to revisit the same person more than once as he distributed objects but move on to next recipient once he saw that the person already had an object. Thus, there was a certain trial and error quality to these matches.

In contrast, there are several examples from Spencer's object-to-slot matches that indicate he was comparing the two sets. Consider the following diary entries:

11/10/01 (33 months): Brian and Spencer were pulling up to the drive-through window at the bank. There were four lanes — there was one truck in each of two lanes; the other two were empty. Spencer remarked, "Look! There's two trucks and two missing parts."

11/30/01 (34 months): While we were waiting to be served at a restaurant, Spencer was playing with a peg game at the table. The game board was a triangle with 15 holes in it arranged in parallel rows. Normally, there are 14 pegs as this leaves one open slot to start the game (the aim of which is to jump all the pegs with another peg). Spencer first dumped all the pegs onto the table and then replaced them one by one. Next, he started looking around on the floor and by sheer luck found a 15th peg. He placed it in the 15th hole and exclaimed, "Look! I found the missing one!"

12/20/01 (35 months): Spencer, Brian, and I were sitting at our dining room table which seats four. Spencer remarked, "Hey, I'm sitting here just like you." I agreed and then he said, “We need another kid in that chair (pointing to the empty chair to his left). We need to get Ben to come and sit in that chair.”

1/3/02 (35 months): Spencer has a puzzle with eight cartoon animals arranged in two rows of four. The heads of each animal are drawn on removable rectangular puzzle pieces. He had worked the puzzle a few days ago, but could not complete it because we were missing the bee’s head. At that time, he matched the other seven heads to the seven bodies and remarked that one was missing. Today, we found the missing bee’s head. Spencer jumped up and down and then ran into the other room where the partially completed puzzle was kept. He put the bee’s head in place and squealed, “Yay! I did it! I did it!” I said, “Yeah, you did it!” He replied, “Look. (pointing to the bee) He’s got a bottom and an up part.” I said, “Right. The up part is called the top. See? Top and bottom.” Spencer replied, “Right. Top and bottom. Now they all got tops and bottoms!”

These entries were among the first and only overt comparisons of sets noted in Spencer’s diary. That these emerged relatively late and within the object-to-slot context is consistent with the idea that object–slot matches form a bridge to notions of numerical equivalence.

In summary, the diary study provides support for three main conclusions. First, children can experience one-to-one correspondence in a wide range of contexts beyond object matches. Second, children may gain access to one-to-one correspondence through socially or linguisitically mediated activities that emphasize local correspondences between items rather than overall numerical equivalence between whole sets. Third, object-to-slot activities, such as completing simple puzzles, may play a pivotal role by acting as a bridge between local pairings and judgments of numerical equivalence. Of course, any conclusions based on these observations must be tempered by the fact that they are based on one child’s development. Still, the observed pattern represents at least one potential pathway. Further research with a larger sample may reveal that this pattern is typical of most children or that several other pathways are possible depending on children’s individual characteristics and learning histories.

4. Early learning is context-specific

The main idea of the previous section was that spontaneous experiences with one-to-one correspondence provide important input for the construction of number concepts. In this section, I consider how this learning might proceed. I will argue that, rather than gradually developing through several stages of partial competence, children may exhibit full-blown competence within very narrow contexts. From this perspective, development would consist of connecting these pockets of complete competence into an abstract structure, rather than gradually assembling this structure from flawed or incomplete pieces. After presenting evidence for this new view from Spencer’s diary, I discuss its relation to the competence–performance distinction.

One general view of learning is that people acquire complex concepts or behaviors in a piecemeal fashion. For example, a basketball player might master free throws, dribbling, and passing separately before moving on to complicated plays that incorporate these skills. Similarly, children learning to read are encouraged to practice decoding-specific sounds, scanning text, and recognizing high frequency words separately from the act of reading. Such teaching methods are grounded in the behaviorist notion of chaining (i.e., the idea that complex behaviors can arise when a series of simple skills are linked together).

The idea of piecemeal learning also appears in Piaget's discussion of the number conservation task. In this task, children were asked to construct an array that matched a large target array. Piaget (1941/1965) described a gradual progression in which children's comparisons rely less and less on perceptual attributes. For example, he observed that children at the earliest stage match the arrays in terms of a global estimate of space (i.e., the overall shape, length, and density). In the next stage, children used more precise qualitative comparisons that took separate dimensions into account. Thus, they had mastered part of the problem (i.e., knowing that line length or density is related to total quantity), but had yet to understand how these two variables were interrelated. Eventually, Piaget argued, children reach a level of coordination that can support lasting numerical equivalence judgments. Although this interpretation does not emphasize mastery of separate components as in the chaining examples above, it seems to share the same general view that development progresses from no competence to partial competence to complete competence. Indeed, much of the controversy surrounding Piaget's research seems rooted in his insistence that early demonstrations of competence were based on partial knowledge rather than the logical ability of interest.

In my observations of Spencer, I noticed a very different pattern of learning — one in which full-blown competence was exhibited in very restricted contexts at the same time that absolutely no competence was exhibited in others. The significance of this pattern is that learning may not be piecemeal at all. Instead, children may master concepts completely in one context after another, only gradually consolidating these diverse pockets of expertise into a coherent conceptual structure.

This pattern was apparent in Spencer's ability to distribute items to recipients in a one-to-one fashion. As noted above, this was the earliest one-to-one behavior to emerge and also the most frequent. However, in most cases, Spencer did not appear to plan these distributions in advance. For example, he might pick up a handful of toys, distribute them until they were all gone, and then go back for more until everyone in the room had one. When Spencer was 21 months old, a situation spontaneously arose that provided a test of his ability to use one-to-one correspondence to represent and match two sets in terms of equivalence. I had just let our two dogs in from the backyard and Spencer was very excited. On a whim, I asked him whether he wanted to give the dogs their treats. I had previously allowed him to distribute treats to the dogs, but I had never let him take the treats out of the box himself. While we went to the kitchen cupboard where the treats were kept, the dogs waited just outside the dining room behind a long baby gate. This arrangement prevented Spencer from being knocked over by the dogs. It also occluded his view of the two dogs while he retrieved the treats.

On the first try, he took exactly two treats and then walked to the dining room where he gave one treat to each dog. On subsequent days over the next week, he either took the correct number of treats or, if he took too many on the first grab, self-corrected by returning treats to the box until he had only two left. Thus, he appeared to not only represent the correct number of recipients in one set, but also matched them with the correct number of items in a another set. This is functionally equivalent to the triad matching task used previously to test numerical equivalence judgments in preschool children (e.g., Mix, 1999; Mix, Huttenlocher, & Levine, 1996). In the simplest version of that task, children see a set of black disks, it is covered with a box, and then they choose the matching set from between two arrays of black dots. Children begin to perform such matches correctly around the third birthday. Spencer exhibited essentially the same competence more than a year earlier.

I should note, however, that his initial success may have been due in part to a verbal prompt. On the first few days he gave treats to the dogs, I was worried that he would impulsively grab as many treats as he could carry and throw them at the dogs. So, I told him, “Just get one treat for Rocky and one for Mushu.” It is possible that he retrieved the correct number of treats because of this prompt rather than his own memory of the dogs. However, there is reason to doubt that he could interpret the prompt. One month after Spencer first gave treats to the dogs, I tested his knowledge of the count words using Wynn’s (1990) “give-a-number” task. I requested six sets of blocks ranging in number from one to three in a random order. Spencer responded incorrectly on every trial. In particular, he gave “3” and “5” blocks on the two requests for one block. Thus, it seems unlikely that he comprehended the prompt in the first place. Although hearing the dogs’ names may still have helped him remember the initial set, the names alone would not tell him the number of treats to retrieve. He apparently made that inference himself.

Over the next 3 weeks, I performed a more rigorous test of this ability. Every day, Spencer was invited to get treats for the dogs. While he retrieved the treats, I did not provide verbal prompts. I also varied the size of the treats each day to see whether Spencer was basing his response on overall amount rather than number. Spencer was extremely accurate. He took exactly two treats on all but one trial, for which he took three treats initially. On that occasion, though I saw that he had taken three treats, I did not attempt to correct him. Even if I inadvertently reacted to his error, it is unlikely that he noticed because he did not make eye contact with me. Instead, he looked down at his hands for a moment and then returned one of the treats himself before moving on to the dogs. In addition to consistently drawing the correct number of treats, Spencer also was highly accurate at distributing them. On every trial, he gave exactly one treat to each dog.

As noted above, the dog treat task tests many of the same numerical abilities as the triad-matching task used to test numerical equivalence judgments in older children. However, whereas Spencer could match equivalent sets when retrieving treats for the dogs, he could not perform the matching task. Following the completion of the dog treat study, I presented the same counting and matching tasks to Spencer used in previous cross-sectional research. These included disks-to-dots matching with the target set hidden (Mix, 1999; Mix et al., 1996), producing numerically equivalent sets (Huttenlocher et al., 1994), give-a-number (Wynn, 1990), and counting 10 objects (Wynn, 1990). Consistent with previous findings from 2.5-year-olds, he was unable to match any of the sets — even when the target numerosity was two. He also performed randomly on both of the conventional counting measures. So, although he was highly proficient at matching to a remembered set in the dog treat situation, this was not reflected in a related experimental task; nor was it due to precocious counting ability.

Furthermore, this ability did not generalize to a related naturalistic situation. When Spencer was 26 months old, after several months of getting treats for the dogs, I devised a similar scenario involving his toy train engines. I chose train engines because those were Spencer's favorite toys and he was extremely interested in them. Also, his trains were from the Thomas the Tank Engine series so each engine had a face and a name. Spencer was quite familiar with the individual characters through books and videos and was proficient at recognizing and naming them all.

For the experimental task, I arranged three of his train engines in a row on the dining room table. Then I said, "Hey, let's get treats for the trains. They like to eat peas. Let's get a pea for each train." Spencer enthusiastically agreed. We went to the kitchen cupboard with the trains out of view and took down a container of freeze-dried peas. Spencer took three peas on the first trial with no prompting. However, when we returned to the dining room, he gave all three peas to each train engine in turn. I repeated the procedure twice more. On the next trial, Spencer took 10 peas. On the last trial, he refused to respond. The next day, I used the same procedure, but this time I started with only two train engines on the table. On the first trial, Spencer took two peas and gave one to each train just as he had with the dog treats. On the second trial, I added a third engine. Spencer took two peas, passed one to each of the three engines and ate the second one himself. After that, he lost interest in the task and refused to participate.

Spencer's performance on the train treat task was mixed in comparison to his performance with the dog treats. On two trials, he did retrieve the correct number of treats. However, because he failed to do so on the remaining trials, it is possible that his correct responses were coincidental. Furthermore, he did not consistently distribute treats to the trains in a one-to-one fashion. In contrast, he always gave one treat to each dog in the dog treat task. This indicates that his competence in the dog treat task was highly encapsulated. It did not generalize at all to similar laboratory tasks and it was barely evident in a related naturalistic task.

What made the dog treat task more accessible than the others? It seemed to provide more scaffolding in three important ways. First, the dogs are extremely salient individuals. They move independently. They have unique names, physical attributes, and personalities. Although this is also true of the train engines to some extent, it is certainly not true of the black disks used in the laboratory tasks. The salience of the dogs as individuals may have made it easier for Spencer to remember them when they were out of view because he could represent them as “Rocky and Mushu” or “the brown dog and the black dog” rather than as “two.” Second, there is a strong functional relation between the sets being matched. In short, the treats are food for the dogs. This relation was mimicked in the train treat task, but of course, trains don’t really eat peas. In the laboratory tasks, there is no natural relation between the disks and the dots except that they look alike. Third, the dogs were active participants in the dog treat task. They helped Spencer distribute the treats one-to-one by taking a treat and then running away to eat it. In contrast, the trains did nothing with the treats. This may have made it easier for Spencer to try many-to-one distributions instead. Obviously, in the laboratory tasks, the disks and dots did nothing to indicate when each individual had been paired up. This may have contributed to the relative difficulty of those tasks.

Further support for these interpretations comes from subsequent observations made in Spencer’s diary. Spencer spontaneously performed several tasks like the dog treat task over the course of the next year. They were similar in the sense that they required him to plan in advance, with one set out of view, the number of items he would need to make a matching set. Then, in view of both sets, he matched the items one-to-one. For example, we were entertaining a dinner guest on our screened porch when Spencer was 29 months old. Spencer spent most of the evening playing in the backyard, where he could not always see us, but returned to the porch every few minutes. On one occasion, he returned with three small sticks in hand. He purposefully distributed the sticks to each of us. We laughed and remarked on his ability to figure out how many sticks to bring. I believe that Spencer overheard us and wanted to impress our dinner guest, because he left again and returned this time with three more sticks. As before, he distributed the sticks to each of us, one-to-one. Like the dog treat task, the individuals in the hidden set were animated, salient individuals with distinctive appearances, personalities, and names. Although the relation between people and sticks was not as functional as for the dogs and treats, it may have been functional to Spencer in that he often plays with sticks in the backyard and may have been enticing us to play with him. Finally, we provided scaffolding for his distributions by taking a stick from him when he offered it.

4.1. A case of competence versus performance?

Patterns of cross-task variability such as the one described here are often interpreted in terms of competence versus performance. The argument goes that once competence has been demonstrated in any context, failures on related tasks are due to irrelevant task demands rather than a lack of ability. For example, even though preschoolers fail Piaget’s number conservation task, it has been argued that they actually possess the ability to conserve because they can demonstrate this on other tasks (e.g., Gelman, 1969, 1972; Mehler & Bever, 1967). The discrepancy between performance on these tasks and Piaget's task is explained in terms of extraneous demands. For example, Piaget's task required children to interpret relational terms, such as "more," "less," and "same as." When quantitative invariance tasks were presented without this linguistic requirement, preschool children demonstrated the ability to match equivalent sets at an earlier age (Braine, 1959; Gelman, 1969, 1972). Thus, it was claimed, children have the necessary competence, but Piaget's task simply failed to tap what they know. The developmental literature is replete with similar examples (see Sophian, 1997).

Some might argue that the present results should be described the same way. Perhaps the dog treat task is a more pure test of Spencer's ability to represent numerosity and detect numerical equivalence than the laboratory tasks. Maybe children fail the laboratory tasks because of theoretically uninteresting demands, such as having to choose the matching set from between two arrays. On this view, Spencer possessed the underlying competence all along but did not demonstrate it consistently in his performance. Whatever changes were observed subsequent to the dog treat study would thus be seen as improved performance and not as conceptual change.

However, it is possible to interpret Spencer's variability on numerical equivalence tasks without distinguishing between competence and performance. Instead, one could say that he was indeed competent within particular contexts (e.g., giving two dogs two treats), but not at all competent in other contexts (e.g., matching two disks to two dots). The idea is that it may be possible to grasp a concept within a particular situation and not grasp it in another. In this view, development would surely consist of conceptual change — first, in terms of stumbling into additional contexts that embody a particular concept and second, in terms of seeing that all these pockets of competence are related by a higher conceptual structure.

Consider the following analogy. Suppose the developing mind is a field that psychologists probe for concepts much like geologists would probe for types of rock — by taking core samples here and there. Let's say that one day a sample revealed the presence of a highly desirable form of rock. Some concurrent and subsequent samples in that field revealed the same rock, but others did not. In the competence–performance view, this would mean that the field "has it" — that the presence of the special rock is certain because there is no other way to explain the samples where it was revealed. The samples that did not reveal the special rock would be dismissed as relatively uninteresting failures — perhaps they didn't use the proper equipment or failed to probe deeply enough.

In the view proposed here, such variability would not be due to differences in probing, but to real differences in the underlying landscape. If we imagine that geological time is on the same scale as developmental time, it would be possible to probe a part of the field in which that special rock has just emerged. Maybe several columns of it have popped up in different locations. Over time, additional areas may form. When there are enough, they may start to overlap and consolidate into a uniform field of such rock.

If at first, one happens to probe in one of the columns, it may seem that the whole field consists of such rock. However, this would really reflect a lucky match between a particular sample and the actual location. For psychologists, this would be a lucky match between a particular task and the naturalistic context in which a child already grasps a concept. As time passes and the desirable rock emerges in new locations, additional probes would be more likely to reveal it. In conceptual development, this would mean that cross-task variability would decrease as children master more related contexts. Eventually, the field could be sampled almost anywhere and the rock would be found. This would be analogous to mature concepts that can be tested a variety of ways.

The interpretation presented here is similar to Nelson's (1999) Functional Core Hypothesis. In this view, children's earliest concepts are embedded in specific event-structures. For example, a child may understand the concept of food only within the script for eating lunch. After children have acquired more of these event-structures, they gradually begin to see commonalities across them. Once this process of abstraction has occurred, children reveal competence on a range of tasks, including those that involve abstract probes. To illustrate, Nelson reported that 5-year-olds listed more foods when they were requested by meal (e.g., "Name some foods you eat at lunchtime.") than they did when the request was more broad (e.g., "Name some foods."). In contrast, 7-year-olds demonstrated the opposite pattern — listing more foods when the request was decontextualized.

The present results, as well as other findings of cross-task variability debated in the numerical development literature, may reflect the same trend. That is, when children reveal numerical competence in a particular task, it may be just that — numerical competence in a particular task (see Thelen & Smith, 1994, for further discussion). What becomes interesting from this perspective are the reasons that certain tasks are more accessible to children early on. Rather than debating about whether children "have" a concept, it becomes more essential to know what about children's individual learning histories and everyday experiences allows them to grasp a concept in one situation but not in another.

4.2. Becoming abstract

If the framework presented so far is correct, and children discover numerical relations within restricted contexts, the next question is how they unite these disparate experiences into higher order concepts. Mix (1999, 2001, 2002) has proposed that number concepts become abstract through the same mechanisms proposed for other concepts — specifically, through experience with the comparison process itself.

Research in other domains has shown that experience matching highly similar items promotes abstraction of particular dimensions (e.g., Klibanoff & Waxman, 1998; Kotovsky & Gentner, 1996; Marzolf & DeLoache, 1994). For example, Kotovsky and Gentner (1996) found that 4-year-olds had great difficulty recognizing the relation between circles that increased in size and squares that increased in darkness. However, when children were trained on same-dimension comparisons (e.g., all sets that increased in size), their performance on cross-dimension comparisons increased significantly. This indicates that when children compare entities that have a great deal of overlap, they extract deeper relations that they can apply in lower similarity comparisons.

There are two conditions that facilitate early comparisons. First, children are more likely to enter into the comparison process in the first place if items share a variety of commonalities (DeLoache, 1989; Gentner & Rattermann, 1991; Smith, 1984). Mix (1999, 2001, 2002) has found similar effects in children's numerical comparisons. For example, when asked to match a numerically equivalent set to a standard, preschoolers are first successful when the sets match on a variety of other dimensions. Even 2.5-year-olds can detect equivalence if the matching sets contain complex objects that share many points of alignment (Mix, 2001).

Second, children's comparisons are facilitated by shared labels (e.g., Rattermann & Gentner, 1998; Smith, 1993; Waxman & Hall, 1993). For example, 21-month-olds in a triad task made more taxonomic matches (cookie–cookie) than thematic matches (cookie–cookie monster) if the items had been given the same nonsense label (Waxman & Hall, 1993). Mix (1999, 2001, 2002) has consistently found that children perform better on numerical equivalence tasks when they know the meanings of the count words for the set sizes involved. In fact, children who fail to demonstrate this level of counting ability only recognized numerical equivalence for sets that were identical or nearly identical. This suggests that numerical comparisons may be mediated by number words just as other comparisons seem to be mediated by object or attribute labels.

5. Conclusions

In research and theorizing about numerical development, there has been a gap in our understanding of the period from infancy to early childhood. This gulf has divided investigators into those who study children and attribute little competence to infants and those who study infants and ascribe little significance to changes in preschool (Mix et al., 2002b). The present paper attempts to bridge this gap by providing detailed information about numerical development in the second and third years of life. The data presented here suggest that children construct number concepts from a variety of one-to-one activities, most of which are embedded in social and linguistic contexts. Furthermore, they fully grasp number concepts within specific event-structures before uniting these structures into more abstract concepts. As more is learned about this age period, it is hoped that continuities will emerge that elucidate both of its endpoints.

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