# CONSTRUCTIVISM: WHICH FORM PROVIDES THE MOST ADEQUATE THEORY OF MATHEMATICS LEARNING?

Paul Ernest

University of Exeter, United Kingdom

Three variants of constructivist learning theory for mathematics are contrasted. These are information processing, radical, and social constructivism. Each position is sketched, and the underlying metaphors of mind and models of the world (with their links to epistemology and ontology) are examined critically. The positions are compared with regard to these issues and their individual-social focus. It is argued that both radical constructivism and social constructivism are adequate learning theories for mathematics education, but that the former over-emphasises the perspective of the individual at the expense of social considerations. The pedagogical implications of each of the different learning theories are also considered.

# 1. INTRODUCTION

One of the central problems facing mathematics education world-wide, both theoretical and practical, is how to best facilitate the learning of mathematics by students. In the context of this problem constructivism has become one of the dominant theoretical positions in mathematics (and science) education. A growing number of mathematics educators identify themselves as constructivist researchers or teachers. Many positive consequences flow from this, for the position embodies a powerful vision of the active and epistemologically empowered learner. But there are also dangers that follow from an over-enthusiastic or uncritical embracing of it, or from a failure to recognise its real limitations. Consequently, in this paper I try to indicate some of the strengths and weaknesses of adopting constructivist views of learning.

However, there is another central problem facing mathematics education world-wide. How can mathematics education best contribute to an emancipatory vision of mathematics education, one which embodies egalitarian values of social justice, and leads to a mathematically literate and empowered critical citizenry? Although I shall not address this question directly, my main point is that if we are to include the learner, teacher, the classroom, the social, cultural and political context, and appropriate aims and values in a theory of mathematics education, a different form of constructivism is needed. It is necessary to include a social dimension in the learning theory adopted, be it some form of constructivism, or not.

In this paper I argue for a social constructivist perspective on the learning of mathematics. My account, acknowledges some of the strengths of radical constructivism, but is also a critique of that position (even more so of 'information-processing constructivism'). Thus this paper is a contribution to a debate that is currently going in the academic community, and which has been represented in print in Ernest (1994a) and Steffe and Gale (1994). However, this debate is not merely academic. World-wide there are teachers and researchers in mathematics education who regard radical constructivism as a panacea to the ills of mathematics education. Part of my purpose in mounting this critique is to signal the danger in neglecting the social dimension of the teaching and learning of mathematics, which often accompanies the adoption of a radical constructivist position. I shall also argue that the most optimistic developments in radical constructivism lie in the development and inclusion of social aspects within it.

# 2. WHAT IS CONSTRUCTIVISM?

There are a number of different forms of constructivism, and it is important to separate them as they vary greatly in terms of theoretical and practical significance. One way of distinguishing them is to search out the underlying principles and metaphors and models for the mind and world.

## 2.1 Information Processing Constructivism

The clearest exposition of constructivism is due to Ernst von Glasersfeld, who distinguishes two basic principles. The first is common to all constructivist positions: "knowledge is not passively received but actively built up by the cognizing subject" (Glasersfeld, 1989: 182). Acceptance of this principle coupled with a rejection of an epistemologically sceptical second principle results in the weak form here called 'information processing' constructivism (also polemically termed 'trivial' constructivism by Glasersfeld).

The information processing paradigm is a broad church which includes the psychology of Ausubel, Anderson, repair theory, and many of the positions adopted by researchers in cognitive science and related psychologies. It is largely based on the metaphor and sometimes the conscious model of the mind as computer. This actively processes information and data, calling up various routines and procedures, and organising the memorization and retrieval of data. It can even be 'heuristically' programmed; that is, modifying its outputs as it learns from experience. The computer metaphor is a very fruitful because it has led to important analyses of human problem solving (Newell and Simon), and to 'buggy' error analysis (Brown and Collins), with important outcomes for the psychology of mathematics education.

As one of its exponents says, alongside cognitive science, information processing theory is "the study of how humans process information, and includes the acquisition, storage and retrieval of knowledge" (Mayer, 1982: 3). The rhetoric of this quotation suggests an underlying computer model of mind. It also indicates the presupposition that some knowledge learned by humans is information which is acquired from the outside world, not constructed within.

Information processing constructivism is thus largely based on the metaphor of mind as computer. The most commonly assumed model of the world is that of absolute physical space populated by material objects (scientific realism). This is the world described by reason and classical science as unproblematic and knowable. Thus information processing constructivism is largely based on a recognisably traditional scientific epistemology.

This position recognises that knowing involves active processing, that it is individual and personal, and that it is based on previously acquired knowledge. This has fairly immediate and beneficial pedagogical implications. Just getting student teachers to realise that learning is constructed, by reflecting on error patterns in mathematics or alternative conceptions in science, represents a significant step forward from the naive transmission view of teaching and learning many student teachers start with. Nor is a passive reception view of learning dead amongst professionals or administrators in education. Many government driven curriculum reforms, in Britain certainly, assume that the central powers can simply transmit their knowledge and plans to teachers who will passively absorb and then implement them in 'delivering the curriculum'. Such conceptions and strategies are deeply embedded in the public consciousness, although it may be no accident that they also serve authoritarian powers. (Freire, 1972; Ernest, 1991).

Thus, an important outcome of this perspective in terms of learning theory (and pedagogy) is that it accounts for student 'error patterns' in mathematics (Ashlock, 1982), and similarly for misconceptions in science. In this respect, some of the work in concept mapping and in the cognitive science approach to research in mathematics education epitomises information processing constructivism (see Resnick and Ford, 1981; Davis, 1984). Likewise, some of the work involving information technology teaching approaches is also representative of the approach (e.g. Spiro et al., 1994). All of this work is valuable if sometimes narrowly technical. But it does not readily lend itself to accommodating the social dimension of mathematics education. I shall not elaborate this criticism here, for it is also directed below at radical constructivism. However it applies just as much to information processing constructivism: more even, for the signs of a modification to include the social dimension evident in some versions or extensions of radical constructivism are not present in this paradigm.[[1]](#footnote-2)

## 2.2 Radical Constructivism

Radical constructivism is based on both the first and second of Glasersfeld's principles. The second is as follows: "the function of cognition is adaptive and serves the organisation of the experiential world, not the discovery of ontological reality." (Glasersfeld, 1989: 182). Consequently, "From an explorer who is condemned to seek 'structural properties' of an inaccessible reality, the experiencing organism now turns into a builder of cognitive structures intended to solve such problems as the organism perceives or conceives." (Glasersfeld, 1983: 50).

The underlying metaphor for the mind or cognizing subject is that of an organism undergoing Darwinian evolution, with its central concept of the 'survival of the fit or fitter'. (This is indicated in Piaget's notion of adaptation to the environment). According to the evolutionary metaphor the cognizing subject is a creature which generates cognitive schemas to guide actions and represent its experiences. These are tested according to how well they 'fit' the world of its experience. The best 'fitting' of the schemas are tentatively adopted and retained as guides to action. Those that do not 'fit' are modified or rejected. Consequently, the radical constructivist cognizing subject might be described as an isolated cognitive alien battling to survive in a hostile landscape.

The underlying model of the world is implicated in that of mind (subjectivism and idealism). It is the cognizer's experiential world, rather like the environment surrounding an animal: experienceable but not ultimately knowable. Together these metaphors are indicative of the epistemology of radical constructivism. It suggests that although we can tentatively come to know the knowledge of others by interpreting their language and actions through our own conceptual constructs, the others have realities that are independent of ours. Indeed it is the realities of others along with our own realities that we humans strive to understand, but we can never take any of these realities as fixed. This brings us closer to a pedagogical constructivism, which as Noddings (1990) implies, adds certain additional values and educational assumptions in order to develop a more elaborated philosophy of education and pedagogical approach.

## 2.3 Social Constructivism

Social constructivism is a more recent development in the psychology of mathematics education, and one immediate problem in describing it is that there are several versions of it. The major distinction between these versions is whether they are individualistic or social in orientation; in particular whether they are Piagetian, or based on a socially orientated theory, such as that of Vygotsky (Ernest 1994d). Piagetian versions of social constructivism occur in the recent work of Wood, Cobb and Yackel (1984), Bauersfeld (1994) and others. These forms of social constructivism draw upon a radical constructivist-like model of mind, so I omit discussion of them in here. What should be said is that they do not have the same epistemological problems arising from a neglect of the social as radical constructivism (Ernest 1994a). It is not yet clear, however, how they accommodate global socio-political problems, addressed e.g. by critical theory.

The second form of social constructivism originated in sociology and philosophy, with inputs also from symbolic interactionism and Soviet psychology. Subsequently it influenced modern developments in social psychology and educational studies, before filtering through to mathematics education (Ernest 1994d). This version of social constructivism is a newer variant of constructivism (Ernest, 1991, In preparation), although in drawing on Vygotskian roots instead of Piagetian, it differs significantly from the other forms discussed above. It is not a form of constructivism per se (i.e. based on Piagetian theory) but a form of social-constructivism.

The problematique of this form of social constructivism for mathematics education may be characterised as twofold. It comprises, first, an attempt to answer the question: how to account for the nature of public mathematical knowledge as socially constructed? In this, it already goes beyond the exclusive focus on learning of the other forms of constructivism considered above. Second, how to give a social constructivist account of the individual's learning and construction of mathematics? Answers to these questions need to accommodate both the personal reconstruction of knowledge, and personal contributions to 'objective' (i.e. socially accepted) mathematical knowledge. An important issue implicated in the second question is that of the centrality of language to knowing and thought. In the light of this characterisation, social constructivist epistemology has two interconnected parts, concerned with the nature and origin of public and private mathematical knowledge, respectively. Beyond this, social constructivism as an educational philosophy can also be distinguished. I shall return to this below.

### Models of Mind, World and Mathematics

The version of social constructivism considered here regards individual subjects and the realm of the social as indissolubly interconnected. Human subjects are formed through their interactions with each other (as well as by their individual processes). Central to this are shared social forms-of-life and located in them, shared language-games (Wittgenstein, 1953).Thus there is no underlying metaphor for the wholly isolated individual mind. Instead, the underlying metaphor is that of conversation, comprising persons in meaningful linguistic and extra-linguistic interaction (Harré, 1979; Rorty, 1979). Thought is understood as internalised conversation (hence the metaphor), and mind is seen as part of a broader context, the 'social construction of meaning' (Bishop, 1985). Mind is viewed as social and conversational because of the following three assumptions. First of all, individual thinking of any complexity originates with and is formed by internalised conversation; second, all subsequent individual thinking is structured and natured by this origin; and third, some mental functioning is collective (e.g. group problem solving). Adopting a Vygotskian perspective means that language and semiotic mediation are accommodated. Through play the basic semiotic fraction of signifier/signified begins to become a powerful factor in the social (and hence personal) construction of meaning, ultimately leading to conscious and voluntarily controlled symbolic thought and action (Vygotsky, 1978).

The social constructivist model of the world is that of social reality, the socially constructed world which creates (and is constrained by) the shared experience of the underlying physical and social worlds. This socially constructed reality is all the time being modified to fit the constraints of ontological reality, as well as to prestructure perceptions of it according to socially accepted assumptions. Consequently human knowledge can never give a 'true picture' of reality (an insight shared with radical constructivism). The key issue that distinguishes it from the experiential world of radical constructivism is that social reality pre-exists any individual who is socialised into accepting a modified local version as his or her worldview, whilst simultaneously participating in the ongoing development of that social reality, which means that its is never static. In addition, although the singular has been used there are of course multiple social realities corresponding to the different context-bound outlooks of different groups of persons, and most persons inhabit and share several of these social realities, as they engage in multiple forms of life.

The second thrust of the social constructivist epistemology is a fallibilist and social account of the nature of mathematical knowledge as a cultural and historical construct. This is based primarily on the work of Wittgenstein and Lakatos. Wittgenstein (1953) offers the basis of a social theory of meaning, knowledge and mathematics resting on dialogical 'language games' embedded in 'forms of life'; i.e. pre-existing social practices which humans share and within which they act and communicate. (These forms of life include those of the classroom, and those of research mathematics.) In this respect the basis is like Habermas' (1981) theory of communicative action, except that the speech community is not idealised.

Lakatos (1976) offers an important but incompletely developed philosophy of mathematics, which provides a dialectical or conversational theory of mathematics adopted by social constructivism. At the heart of this is his heuristic or Logic of Mathematical Discovery, which is a dialectical theory of the history, methodology and philosophy of mathematics. This is elaborated into a social constructivist dialogical account of mathematics (Ernest 1994b, In preparation)

The social constructivist account interrelates the individual and public aspects of mathematical knowledge production, communication and warranting, and offers parallel accounts in both the educational and research spheres. Within the contexts of mathematics education individuals use their personal knowledge of mathematics (and mathematics education) to direct and control mathematics learning conversation both (a) to present mathematical knowledge to learners directly or indirectly (i.e. teaching), and (b) to participate in the dialectical process of criticism and warranting of others' mathematical knowledge claims (i.e. assessment of learning).

Similarly, within the contexts of professional research mathematics, individuals use their personal knowledge both (a) to construct mathematical knowledge claims (possibly jointly with others), and (b) to participate in the dialectical process of criticism and warranting of others' mathematical knowledge claims. In each case the individual mathematician's symbolic productions are (or are part of) one of the voices in the warranting conversation.

There is an overall productive/reproductive cycle by means of which mathematical knowledge is created, transmitted and reproduced. Public mathematical knowledge is transmitted in the form of (accepted) texts, which are transformed for educational purposes. Private mathematical knowledge consists in individuals' capacities which are themselves transformed through education. The negotiations through which both public and private mathematical knowledge are transformed are conversational.

Thus as well as providing the central metaphor that underpins the social constructivist model of mind conversation is central to the pedagogical or educational philosophy associated with the position. Conversation plays an essential role in the teaching and learning of mathematics, because individual learners develop personal knowledge of language, mathematics and logic through prolonged participation in socially situated conversations of varying types. In the context of mathematics education teachers structure mathematical conversations on the basis of texts and their own knowledge in order to communicate mathematical knowledge to learners. However this is necessary but not sufficient for such knowledge to be learned. Sustained two-way participation in such conversations is also necessary to generate, test, correct and validate personal mathematical knowledge. The acquisition and use of subjective knowledge of mathematics by individuals are irrevocably interwoven. For only through utterance and performance are the individual construals made public and confronted with alternatives, extensions, corrections or corroboration.

More complete details of this model are provided elsewhere (Ernest, 1991, 1994). The key issue in the present context is that both educational and disciplinary knowledge of mathematics are fallible social constructions from the perspective of social constructivism. By introducing the social dimensions of knowledge warranting, social constructivism inescapably introduces power and social interests into both the domains of schooling and research, something that most other constructivist epistemologies do not (although other social philosophies such as the Critical Theory of the Frankfurt school most notably do.)

### Social Constructivism as Part of an Educational Philosophy

The account I have given is of social constructivism as an epistemology and learning theory. As an educational philosophy more is involved, including the assumption of a view of society and the goals of education, and is thus based on a set of values. In Ernest (1991) I argue that social constructivism only becomes a fully explicit educational philosophy through co-ordination with a number of other assumptions and values. The overall position that I propose is termed the Public Educator ideology, drawing on Williams’ (1961) analysis of the ideologies of different social groupings in the history of British social thought. This position is based on a particular set of values, comprising social justice, liberty, equality, humanitarianism, social awareness and engagement, and critical citizenship. It is also based on a view of society as an inequitable hierarchy, which is need of reform.

In many ways this position resembles the Critical Theory of Marcuse (1964), Habermas (1971) and the Frankfurt School. The interest is emancipatory: to reform society through schooling towards greater equity and social justice. The critical orientation has been most fully explored in mathematics education by Skovsmose (1985, 1994) and Mellin-Olsen (1987), and the social constructivism philosophy that I am proposing is consistent with these positions.

The combination of the social constructivist epistemology coupled with Public Educator values has important implications:

* for the didactics of mathematics and educational issues including those of mathematics and gender, race and multiculture, since it recognises the social import and value-laden nature of mathematics;
* for pedagogy, by supporting fully investigational and problem solving approaches as paralleling the means by which mathematical knowledge is generated and as a means of promoting epistemological empowerment and critical mathematical citizenship;
* for its enabling of challenges to hierarchical views of mathematics, learning and ability due to a rejection of fixed and objective epistemological hierarchies.

The aim of this perspective is that of critical mathematics education: to educate confident problem posers and solvers able to critically evaluate the social uses of mathematics.

# 3. COMPARISON

An examination of the underlying metaphors for mind and world models of the various forms of constructivism reveals significant differences between them, as Table 1 shows.

## Table 1: Constructivist Metaphors for Mind and World-Models

|  |  |  |
| --- | --- | --- |
| **TYPE OF CONSTRUCTIVISM** | **METAPHOR FOR THE MIND** | **MODEL OF THE WORLD** |
| **INFORMATION-PROCESSING CONSTRUCTIVISM** | Computer, unfeeling thinking machine | Absolute space with physical objects |
| **RADICAL CONSTRUCTIVISM** | Evolving, adapting, isolated biological organism | Subject's private domain of experience |
| **SOCIAL CONSTRUCTIVISM** | Persons in conversation | Socially constructed, shared social worlds  |

An important feature is the varying complexity of the underlying metaphors of mind. Information processing constructivism sees mind as an active but inanimate machine, the computer. Radical constructivism sees mind as an evolving and adapting organism, like an animal, alive but not necessarily human. Finally the metaphor of mind of social constructivism is that of persons in conversation, according mind human status, and recognising that social and linguistic contexts are definitionally essential to the human nature of mind. To paraphrase Rom Harré, for purely scientific reasons alone (never mind moral purposes) it is best to treat subjects as persons, and minds as human. Only then can we anticipate the richness of human thought, feeling, values, reflection, planning, purposes and goals. Thus my criticism is that of the three perspectives considered, only the latter immediately accords embodied human minds full personhood. (But note that this strength can be shared by other perspectives mentioned below.)

Another important aspect is the model of the world, which indicates the underlying view of reality and knowledge. Information processing constructivism brings with it an absolutist epistemology, the assumption that ultimate knowledge in mathematics, is possible. It locates the epistemological problematic exclusively in the object to be known, that is in the mind of the learner, and sees no unavoidable need for reflexivity or doubts about the researcher's constitutive role in knowledge and meaning making.

In contrast, in keeping with recent thought in the philosophy of mathematics, science and social sciences, radical and social constructivism both have fallibilist epistemologies which reject the possibility of absolute knowledge. They problematise the relationship between the knower and the known, and accept that no certain knowledge (including mathematical knowledge) is attainable by humans. There is thus a humility with regard to epistemology, knowledge and the results of research. This is important, as interdisciplinary modern thought has moved away from absolutist conceptions in all fields of knowledge .

Table 2 contrasts the epistemologies of the positions, as well as the focus in terms of the individual-social distinction.

## Table 2: Classification of Types of Constructivism by Epistemology and Focus

|  |  |  |
| --- | --- | --- |
|  | **ABSOLUTIST EPISTEMOLOGY** | **FALLIBILIST EPISTEMOLOGY** |
| **INDIVIDUAL FOCUS ALONE** | INFORMATION-PROCESSING CONSTRUCTIVISM | RADICAL CONSTRUCTIVISM |
| **DUAL SOCIAL AND INDIVIDUAL FOCUS** |  | SOCIAL CONSTRUCTIVISM |

Radical constructivism has on the face of it an exclusively individualistic focus, whilst social constructivism sees individuals as irreducibly implicated in and constituted by the social. Once again, I must emphasise that social constructivism is not unique in these valuable characteristics; it is just the only one of the three perspectives discussed here that has them.

In view of the telling criticism of the information processing version of constructivism offered above, I shall discount it, and concentrate on what I take to be the more serious candidates for a theory of learning of mathematics: radical and social constructivism. Thus I begin with a critical review of radical constructivism.

# 4. EVALUATING RADICAL CONSTRUCTIVISM

The individualistic emphasis of radical constructivism leads to some significant weaknesses. Its cognizing subject appears to be near-hermetically sealed in a privately constructed experiential world of its own. Its representations of the world and indeed of other human beings are personal and idiosyncratic. Indeed, the construal of other persons is driven by whatever representations best fit the cognizing subject's needs and purposes. Such a view makes it hard to establish a social basis for interpersonal communication, for shared feelings and concerns, let alone for shared values. By being based on the underlying evolutionary metaphor for the mind there is a danger that interpersonal relations are seen as nothing but competitive, a version of the 'law of the jungle'. After all, this is but another way of phrasing 'the survival of the fit', or even the competitive conservative 'market-place' ideology. Yet society and its functions, in particular education, depend on articulated and shared sets of concerns and values. The secondary role accorded to the realm of the social and other persons might be a serious weakness for a theory of mathematics education if it is to get beyond the individual learner to interpersonal relations, and the social context of schooling, let alone engage with issues of values, power and politics.

As radical constructivism spreads in popularity, a danger is that it can lead to an overly child-centred, romantic progressivism. Constructivism, conceived in a loose and emotive way, can become associated with a sentimental view of the child (Walkerdine, 1984), a mythologising of childhood, and an over-shielding of the child from social influence, from the 'nasty' realities of the world. This romanticism, as part of the progressive teaching ideology, sanctions anything the child does as expressions of its individual creativity, and naively assumes that the child can discover much of conventional school knowledge on its own. Discovery Learning in the 1960s onwards was often bound up with a romanticism which in the end was not wholly productive for learners, and we must guard against constructivism becoming identified with this position. Whilst there is a need to let learners construct their own meanings, the teacher (and peers) must interact with learners to negotiate a passage towards socially accepted knowledge (Mellin-Olsen, 1987).

This point highlights a significant weakness of radical constructivism: its epistemology. Much of modern philosophy, especially interdisciplinary studies of science and knowledge, whilst rejecting traditional objectivist accounts view knowledge as constitutively social. Ultimately, it is difficult without contrived and contorted thought for radical constructivism to view knowledge as anything but subjective. But in education, above all places, it is clear that we are offering learners socially accepted, constituted and represented knowledge. Thus the subjectivist approach of radical constructivism towards knowledge and epistemology must be regarded as a significant weakness. It does entail a simplified individualistic cognitive theory of mind, which as well as omitting the vital social and cultural dimensions, is theoretically problematic (Ernest 1993).

Of course these is an legitimate reply to some of these criticisms, which although not accommodating the epistemological criticisms fully does address those of education. The radical constructivist epistemology itself does not include a social dimension, and thus does not immediately suggest a consistent position in terms of the social, values, and so on. But just as in the case of social constructivism, a set of values and assumptions about the aims of education and the nature of society can be added to it, to give an overall educational philosophy. Not only is this a theoretical possibility, but it is to be found in some of the recent publications in the field. Radical constructivism is a rich theory which is giving rise to a whole body of fruitful and innovative research into the learning of mathematics (Glasersfeld 1991, 1994, Bauersfeld 1994, Steffe and Gale 1994, Wood, Cobb and Yackel 1994). Authors including these are grappling with the problem of how to accommodate the social in constructivism. Indeed, some of them have been developing form of social constructivism building on Piagetian or radical constructivist basis for many years (e.g. Bauersfeld 1980, 1994). There is every indication that some of the problems of earlier versions of radical constructivism are being overcome, and that overall it constitutes a progressive research programme, in Lakatos’ (1978) terms.

## Implications for Practice

Ultimately, the import of an educational epistemology or philosophy concerns its implications for practice, notably in pedagogy. However, there is little in any pedagogy that is either wholly necessitated or wholly ruled out by an epistemology or even an educational philosophy. A pedagogy is based on a set of values, assumptions and an epistemology. But it remains a theory of techniques for achieving the ends of 'communicating' or offering selected knowledge or experiences to learners in a way consistent with these values and assumptions. Thus any so-called ‘implications’ for practice are not logical deductions, but consistent approaches that seem to ‘resonate’ or cohere with the background educational philosophy. With this warning, it can be said that the three groups of constructivist perspectives suggest the following valuable pedagogical implications. These include, first of all, the need and value for:

* Sensitivity towards and attentiveness to the learner's previous constructions;
* Diagnostic teaching attempting to remedy learner errors and misconceptions, with cognitive conflict techniques as part of this;
* Attention to meta-cognition and strategic self-regulation by learners;
* The use of multiple representations of mathematical concepts.

Beyond these pedagogical emphases, a number of stronger implications follow from radical and social constructivism, including an emphasis on:

* The subjective construction of meaning: the ability to construct, call up, and enter into the resultant personally imagined 'math-worlds';
* A concern with learner cognitions, beliefs, and conceptions of knowledge;
* A similar concern with teacher knowledge, beliefs, and personal theories and conceptions about subject matter, teaching and learning;
* Knowledge as a whole is problematised, not just the learner's subjective knowledge;
* Methodological approaches are required to be much more circumspect and reflexive: there is no 'royal road' to truth;
* The import of values, purposes and goals underpinning the processes of mathematics education as an intentional activity
* Awareness of the importance of goals for the learner, and the dichotomy between learner and teacher goals;

Social constructivism (both varieties) also picks out the following as features of significance for the mathematics classroom:

* The linguistic basis of mathematical knowledge, and in particular, the role of its special symbolism in mathematics;
* An awareness of the social construction of knowledge; suggesting a pedagogical emphasis on discussion, collaboration, negotiation and shared meanings;
* The social processes involved in the determination, construction and negotiation of mathematical concepts, methods, symbolism, arguments and results;
* The social processes involved in the warranting of knowledge (the assessment of learning);
* Awareness of the importance of social contexts, such as in the difference between folk or street mathematics and school mathematics
* The social and cultural context within which all mathematics occurs, including interpersonal relationships, social institutions and power relations;
* The historico-cultural context of mathematics, the sources and uses of the artefacts, tools and concepts involved.

# 5. CONCLUSION

In this paper I have offered a critique of information processing constructivism, and suggested certain problems in the adoption of radical constructivism as a theory of learning (while stressing that parts are valuable, and that important extensions are underway). Constructivism may help to reconceptualize the teaching of mathematics, but it does not strictly imply or disqualify any teaching approach. Rote learning, drill and practice, and passive listening to lectures can, as they always have, give rise to learning. Active learning can be mental, and so visible inactivity on the part of the learner is irrelevant. Some teaching techniques may possibly be more or less efficient than others, but the constructivist view of learning does not rule out any teaching techniques in principle. Nor does it equate to the 'discovery method' or problem-solving teaching approaches (Goldin, 1990). (Having said this, I have offered some possible pedagogical emphases that fit with each of the three forms of constructivism considered here.)

In addition, radical constructivism if overly individualistic can be a distraction from social and political goals in education, from the goal of empowering a mathematically literate citizenry, able to use their skills productively and critically. The individualistic forms of constructivism focus attention away from such goals. The combination of social constructivism with appropriate social justice values and emancipatory aims for mathematics education can offer such a radical solution (Ernest, 1991). Of course the new forms of social constructivism based on radical constructivism may begin to address such goals, but they do not do so yet. Finally, there is growing recognition that the form, genesis and justification of knowledge are constitutively social, and that any overly individualistic or subjective account of it misses out on its essential characteristics. Radical constructivism suffers from this weakness, whereas at the heart of social constructivism is the recognition that knowledge is socially constructed (Ernest, 1991, In preparation).

However I am not suggesting that social constructivism offers a unique panacea. Indeed some of its strengths can also be found in Critical Theory, social psychology, sociolinguistics, post-structuralism, symbolic interactionism, activity theory, and the sociology of knowledge, although none offers a pedagogy.

Although the various versions of social constructivism currently look set to overcome the problems indicated above, they too will meet new problems and ultimately will need to be modified or rejected in favour of some other perspective, perhaps from the fields listed above. The new epistemological humility that follows from both radical and social constructivism tells us that however attached we may become to our theories and solutions, none can survive in the long-term. Our theories are tools, only to be used for as long as they remain useful, and technology always outgrows itself, in the face of human needs and ingenuity.

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University of Exeter,

School of Education,

Exeter, EX1 2LU

United Kingdom.

1. Unless one includes the situated cognition movement, reflected in the work of Brown, Collins and Duguid (1989) and others. [↑](#footnote-ref-2)