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REVIEW

Persuasive Pedagogy: A New Paradigm for Mathematics Education

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Abstract Mathematics teachers face a myriad of instructional obstacles. Since the early 1990s, mathematics education researchers have proposed the use of constructivist practices to counteract these ever-prevalent obstacles. While we do give credit to the choices of instructional activities the constructivist paradigm promotes, there are problems with its use as the foundation of mathematics pedagogy (e.g., Phillips, Educational Researcher 24: 5–12 1995; Simon, Journal for Research in Mathematics Education 26: 114–145 1995). In this paper, we will analyze and review the literature pertaining to the conceptual tenets and operational practices of constructivism, and the viability of these practices for meeting the professional teaching standards proposed by the National Council of Teachers of Mathematics (NCTM; 2000). We will then review the literature pertaining to a paradigm of teaching that may be more applicable, that of persuasive pedagogical practices, and the ways in which these practices can differentially meet the goals of the mathematics standards. The differences between constructivism and persuasive pedagogy lead us to believe that the adoption of the theory of teaching as persuasion, or persuasive pedagogy, may be more appropriate for learning mathematics and the identification and correction of misconceptions. Further, these pedagogical practices correspond with suggestions for mathematical discourse provided by NCTM (2000).

Keywords Persuasive pedagogy · Mathematics education · Mathematics pedagogy

Mathematics is an essential piece of the school curriculum. Not only is it a major emphasis in standardized testing (No Child Left Behind Act, 2001), but it is also a gateway into a college future. At a more basic level, mathematics is helpful in everyday life, for activities such as creating a budget, understanding the numerical data presented by the media, making

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purchases, and other day-to-day encounters. Because of the monumental importance of mathematics outside of the classroom, there is clearly a need for students to learn and have a deep understanding of the mathematical topics taught in the classroom. Despite the importance of mathematics teaching, anecdotal and experimental evidence leads us to believe deep understanding and learning of mathematics may not be happening. In national measures, gains in tests scores may result only from the fact that the test has been in place for a period of time (Abrams *et al*. 2003), and not because of increased learning (Cizek 2001). To counteract this, new conceptualizations of pedagogical practices may be needed to increase student mathematical learning.

In mathematics education, many researchers propose to increase student learning through teaching using methods that appear, on the surface, to align with the framework of radical constructivism. This perspective states that each student should be led through a series of mathematical experiences in order to construct a personal representation of mathematics and how it works (Steffe and Kieren 1994). This framework for teaching demands much content knowledge from the teacher, content knowledge that is not always present.

The adoption of constructivism as the theoretical underpinning for learning and pedagogy in mathematics classes provides no mechanism for teachers to explicitly acknowledge and correct the implicit beliefs held by individuals about the knowledge they have gained and that they are trying to learn, in the way that they are defined by conceptual change researchers (e.g., Carey 1985; Chi 2005; Murphy and Mason 2006). Changes in beliefs are deeply entwined with changes in knowledge (Vosniadou 2003). Very often, individuals who are learning new information hold onto entrenched beliefs that arise from incorrect information or perceptions (Vosniadou 1992). They create what are known as misconceptions by changing their mental models via accommodation (Piaget 1955) or radical restructuring (Rumelhart 1980) in such a way that they erroneously reconcile these entrenched naïve beliefs with new information (Vosniadou 1992, 1994). These misconceptions occur because students use their “common sense ideas” and beliefs about erroneous perceptions or information to interpret processes about which they have little knowledge (Chi 2005). Beliefs students have about mathematics and mathematical learning may hinder their abilities to learn the subject (Mason 2003). Thus, in order to truly impact the mathematical learning of students, teachers must use pedagogical practices and a corresponding theoretical framework that cause students to question their intuitive beliefs (Vosniadou 1994).

In other words, mathematics educators must subscribe to a theoretical framework that allows them to craft a message in such a way that the characteristics of that message confront students’ naïve theories (Alexander *et al*. 1998; Guzzetti and Hynd 1998). Because constructivism is constrained by the social and physical environment (von Glasersfeld 1990), students may not construct knowledge that is accepted as correct, leading to inappropriate knowledge constructions that do not fit with what is accepted by the mathematics community (Lerman 1996). Teachers may not be effective in changing the underlying beliefs students hold if they rely on students’ abilities to use their own logic to construct responses as advocated by the constructivist paradigm (e.g., Dole and Sinatra 1998; Steffe and D’Ambrosio 1995) because students may rely on the erroneous beliefs they are trying to change or eliminate to construct those responses (Chi 2005).

In response to the constructivist paradigm, offered mainly as a post-epistemological explanation of conceptual development based upon a separation from objective truth and reality (Noddings 1990; von Glasersfeld 1990), we suggest mathematics educators adhere instead to a theoretical framework referred to as persuasive pedagogy. This theoretical framework details the teaching of mathematics for learning and comprehension in a way that more closely matches the pedagogy being used by innovative and effective mathematics teachers. The theoretical framework of persuasive pedagogy facilitates learning experiences that promote problem solving, reasoning and proof, communication, links to prior knowledge, and multiple representations of the information mathematics educators often use in their classroom teaching. By integrating problem solving experiences, educators can stimulate further mathematical application and learning. By requiring reasoning and proof to justify beliefs, students have the opportunity to develop and evaluate conjectures about mathematics. Persuasion forces the students to connect with prior knowledge and create multiple representations that not only strengthens the learning experiences, but also increase retention and application. This type of pedagogy has been shown to be effective in other content areas (e.g., Alexander *et al*. 2002).

One reason persuasive pedagogy has value in the mathematics classroom is that the practices of justification, argumentation, and discussion are fundamental aspects of this teaching pedagogy that not only confront student misconceptions, but also student beliefs about knowledge. This is important because merely addressing misconceptions does not modify the underlying beliefs that are used in constructing meaning (Vosniadou 1992, 1994, 2003; Chi 2005). Teachers who incorporate persuasive practices into their teaching may find themselves and their students better equipped to handle the misconceptions that are common in the learning of mathematics. In other proposed and implemented teaching theories such as constructivism, methods for correcting misconceptions are not as explicit and direct, thus making them less effective.

Our purpose is to investigate the theoretical underpinnings of persuasive pedagogy as they may apply in the mathematics classroom as a viable alternative to the constructivist framework adopted by the mathematics education community. In this paper, we describe some of the most advocated theories of how mathematics should be taught (i.e., radical and social constructivism). After critiquing these theories and methods, we turn to persuasive pedagogy and how it may offer solutions to problems identified with adopting a constructivist framework underlying mathematics pedagogy. An example of effective mathematics teaching is provided at the end of this review.

Radical Constructivism

In response to the objective-driven, didactic teaching methods traditionally used in mathematics classrooms (Hiebert *et al*. 2003; Stigler and Hiebert 1999, 2004; Wu 1999), some researchers turned to a more cognitive model focusing on conceptual change (Confrey 1990). Researchers felt that purely fact-based mathematics lessons promoting drill-and-practice pedagogy did not reflect the personal experiences of the learners (Cobb 1994) and saw them to be too opaque or without character (Steffe 1990) in that it did not reflect the value of the individual's constructions. They believed, instead, that the mathematical abstractions of the students were more important than the mathematical abstractions of the teachers in understanding concepts (Steffe and Kieren 1994).

Researchers who labeled these traditional practices insufficient for mathematics education have recently been drawn to a perspective of teaching called radical constructivism (Steffe and Kieren 1994). This framework describes learning as the act of building up cognitive structures based upon one's own views of reality. Radical constructivists do not deny that one, objective reality exists, but claim that there is no way a person can ever know what it is (Goldin 1990; Hardy and Taylor 1997). In fact, as von Glasersfeld (1990) points out, a person has no way of knowing the objective truth because "truth" is seen through a person's own perspective. He states that, "[t]he expression 'shared meaning' is a deceptive fiction because sameness can never be ascertained" (von Glasersfeld 2006, p. 69). Objective reality lies beyond our experience and our cognitive ability to experience it. Instead, our senses and previous conceptual adaptations help us to obtain knowledge. This knowledge is built based upon reoccurring experiences by the participant (von Glasersfeld 1989).

Because our conceptions of reality are built in different ways (Steffe and Kieren 1994), radical constructivists define knowledge differently than what would be thought of by those adhering to traditional epistemic stances. They state that "…knowledge consists of mental constructs which have satisfied the constraints of objective reality" (Hardy and Taylor 1997, p. 137). Because their conception of knowledge is different, their definition of learning must also be different. Learning is defined as the neutralization of a disturbance brought by sensory perceptions (Hardy and Taylor 1997). This neutralization is accomplished when a person reorganizes their model of experience and the activities associated with that experience. In other words, constructivists see knowledge as how a person individually makes sense of their experiences within their interpretation of reality (von Glasersfeld 1990). When a person experiences a situation in which prior understanding conflicts with their current sensory experience, the changing of their understanding is learning.

Thus, for a radical constructivist teaching mathematics, knowledge consists of the model of experience and its associated mathematical activities. Radical constructivists believe that the human mind can only know what it has constructed (von Glasersfeld 1990), and that knowledge is not passively received. Rather, new mathematical knowledge is made by the action of the knower. One wonders, then, whether or not a learner could choose to construct any model of reality they wish. Hardy and Taylor (1997) debunk this notion by stating that models of reality constructed by an individual are only considered knowledge when they explain perceived reality. When the mathematical model a student has constructed fails to work, it is replaced. This replacement of failed models by newly constructed models that better explain perceived reality is defined as learning.

Social Constructivism

Radical constructivism is seen to have many faults by a number of researchers (Goldin 1990; Cobb and Yackel 1996; Garrison 1997). For example, Garrison (1997) disagrees with von Glasersfeld's notion that people construct their own reality because this individual reality presents only a person's subjective view. Radical constructivists claim that only nature plays a role in the development of conceptions (von Glasersfeld 1996); however, social constructivists believe that society and social situations facilitate conceptual development (Confrey 1990; Garrison 1997). Garrison posits that an individual's subjective reality is built through a shared reality that everyone experiences. Although radical constructivism does not deny that a social component influences the building of a shared reality (Hardy and Taylor 1997), they do not explain its influence on individual learning.

Social constructivists, in contrast to radical constructivists, do explain the influence that a shared reality has on learning. For example, Cobb and Yackel (1996) completed a design experiment to teach second grade students mathematical concepts in a radical constructivist format where each student was encouraged to construct his own meaning of the concept. They found that this theory did not work in a classroom, for two reasons. First, students needed to be able to explain their interpretations to other students and the teacher as part of normal classroom procedures. Second, they found that the academic beliefs of an individual and the social norms of a classroom-learning situation were reflexively related to one another. The discussion of concepts greatly influenced the students' mathematical development.

Discussion of ideas is critical to learning, for two reasons. First, without language, meanings cannot be made because there would be no mechanism for mutual understanding (Confrey 1990; Garrison 1997). Making meaning is a property of social behavior, then, because we are dependent upon a shared language for the construction of meaning (Confrey 1990; Garrison 1997). Second, context and social dialog play a role in the convergence of meaning (Steffe 1990; Loong 1998) because we develop the skills of interpretation and the ability to construct meaning through socialization (Resnick 1989). Social constructivism considers the contextual nature of a learning situation, stating that people perceive their experiences as it makes sense to them within a given social context. As Garner and Hansis (1994) put it, "…people are not governed by recognition of some invariant objective reality; rather, they construct the events that unfold around them, sometimes using stereotypes and other schemas without awareness" (p. 68).

In theory, the practice of social constructivism in math classrooms sounds enticing because teachers want to promote student discussion in order to enable better learning (e.g., The National Council of Teachers of Mathematics 2000; Waggoner *et al*. 1995). Researchers have long found that the presence of effective discussions about content promote increased learning (e.g., Chinn and Anderson 1998; Commeyras 1993; Murphy *et al*. 2009), and these findings are similar in mathematics contexts (Loong 1998). While Cobb and Yackel (1996) found that students increased their mathematical understanding by using a constructivist framework for discussions, they do not offer any insights into what aspects of the discussion increased students' learning (see also Noddings 1990; Simon 1995) nor are we assured that misconceptions can be identified and corrected. Social constructivism moves in the right direction in that it allows the teacher the ability to steer students toward accepted and shared understandings. However, as will be pointed out in later sections, the adoption of this paradigm does not adequately represent or describe the actions of the teacher in a mathematics classroom, nor does it alleviate the problem of students relying on prior erroneous beliefs to create mathematical understandings.

Practices of Constructivism in the Mathematics Classroom

Mathematics teachers who intend to teach in the constructivist paradigm inevitably must consider the social aspects of the paradigm, as they foster a learning environment that is based on the interaction of their students. In addition, they work under the assumption that there is a reality of mathematical concepts and rules that exist outside the students' minds that they would like their students to learn. A successful mathematics classroom in the constructivist paradigm will incorporate many mathematical ideas that are accepted by the class because of shared events they have interpreted together as implying certain mathematical rules or laws.

Description of What Constructivist Teaching is Not

While it is difficult to perfectly describe constructivism due to the diverse perspectives of its subscribers and the contextual nature of learning, let us transcend these discrepancies and look at what constructivism is not. Because of the individual nature of constructivism, it cannot be defined as a specific pedagogy. In fact, Simon (1995) quotes Bauersfeld (1995) as saying that once constructivism is operationalized, it is no longer constructivism. At minimum, constructivists can define what its teaching is not. As well as not being a specific pedagogy, constructivism is also not discovery learning, the method of teaching in which students are left to "happen upon" valuable knowledge. Constructivists are well aware that the mathematical knowledge that elementary students are expected to learn is the product of hundreds of years of great thinkers, and will not be discovered by a 7-year-old. But, constructivist teaching is also not done in the traditional teaching style of lecture.

Tenets of Constructivist Teaching

If we once again transcend the diverse interpretations of constructivism, it can be described as an informed exploration of ideas (whether they are mathematical or not). For example, Simon (1995) and Steffe and D'Ambrosio (1995) characterize constructivism as reflective inquiry of the subject that is accomplished by thoughtful design based on teacher's understandings of student comprehension. Constructivist teaching encourages reflection by both teachers and students and promotes interactive mathematical communication. In essence, the hallmark of constructivist teaching is that activities are thoughtfully chosen by the teacher to allow the students to construct their knowledge through interaction with others and ideas. In mathematics classrooms, discussion and reflection are key elements of this learning process (The National Council of Teachers of Mathematics 2000).

Although the research is clear that constructivism does not consist of a specified set of pedagogical practices, researchers do give examples of lesson planning and lessons. A key point is for teachers to spend time reflecting on student knowledge. While constructivist teachers understand that the individual nature of knowledge will not allow them a perfect understanding of what students know about a topic, through reflection on interactions with students they can hypothesize what the student knows and what sort of learning experiences would enable the student to move to the next level of knowledge or to learn another important concept or process. Teachers can then plan their lessons accordingly. Simon (1995) calls these reflective lesson plans hypothetical learning trajectories. It is also vital for discussion to be a significant part of the teaching process so that the teacher can continually gauge whether the lesson is heading in an appropriate direction and if the students are constructing viable knowledge from the activities. By asking the students to take responsibility for their learning (through discussion and problem solving) they learn more and participate in the needed reflection to grasp the ideas of the lesson.

Constructivism and the NCTM Standards

Many of the constructivist practices correspond with the National Council of Teachers of Mathematics (NCTM) Principles and Standards (2000). In fact, the Journal for Research in Mathematics and Education (published by the NCTM) propagated a constructivist approach to mathematics in their 1990 publication. These standards and practices align such that learning in math classes happens through meaningful interactions with real problems and students are encouraged to reason through their solutions and provide overt representations of their constructions. Much of the student's knowledge construction occurs when encouraged to communicate their ideas and methods for solving. This communication allows communities of students to create mathematical ideas that are "taken-as-shared." This would imply that proof of an objective truth is no longer necessary for those ideas because the community has accepted their validity. Teacher knowledge of student prior experiences facilitates the development of trajectory plans that help elaborate on what the student already knows. Experience with different representations of information enables students to interact more completely with a concept, thus increasing the probability that they will construct useful ideas about the concept.

In a constructivist classroom, the role of the student is to construct knowledge. Through interactions with mathematical manipulatives, planned activities, and procedures, it is understood by constructivists that students will construct a personally meaningful system of solutions for the topic. Constructivism expects that what students experience in the classroom will become their representation of the phenomenon. Prior knowledge is changed based upon the lack of correspondence it has with new constructions (Hardy and Taylor 1997).

Alternate Conceptions in Constructivism

One area where subscribing to either a radical or social constructivist paradigm may fail to help teachers is when a student gives evidence of believing a misconception. Because constructivists do not believe an objective reality is knowable, issues of right and wrong become somewhat inappropriate. It is clear in math, however, that some points must be agreed upon for further progress to be made. The assumption in constructivism is that if a student constructs a conception, then they find it to be adaptive to their experiences in their environment. Yet with further experience, the student may discover that the original conception is not as useful as before. Because of new constraints, what was previously adaptive is now maladaptive (Simon 1995). When a student realizes that the conception is no longer viable in their experienced world, they experience disequilibrium. These internal perturbations are neutralized through further reflection and experience leading to the formation of more appropriate conceptions. To encourage a student to learn more, teachers need to create situations where students experience these perturbations, and are then provided with the time and opportunity to neutralize them. Since the teacher must infer what the student understands, this process can be challenging. It calls for a teacher who will invest the time and energy into communicating with his or her students, and will then reflect on the most likely way to either create or resolve these maladaptive conceptual ideas through experiences that more directly address the underlying problems (Davis *et al*. 1990).

Persuasive Pedagogy

Due to the uncertainty and individualistic perspectives and underlying beliefs of students about knowledge inherent in constructivist teaching, we propose that a viable solution to the problems in current math education is in the model of persuasive pedagogy (Murphy 2001; Sinatra 2005). Persuasive pedagogy is a paradigm of teaching that, like social constructivism, sees learning as a process of shared understanding. Learning is not just a transmission of information, but a dialog between student and teacher to come to a common understanding of the topic at hand (Murphy 2001). The way that persuasive pedagogy differs from constructivism is in its conceptualization of student roles and their prior knowledge.

In the teaching as persuasion paradigm (Murphy 2001), students and teachers acknowledge that there is often no one correct view in complex problems and that multiple perspectives are worthy of investigation. Persuasive pedagogy sees students as active participants in their knowledge making process by using methods of argumentation and justification (Murphy 2001). Student prior knowledge and beliefs are essential to the teaching–learning process. Experienced teachers will quickly admit that students are not blank slates, but instead enter the classroom with preconceived notions and perceptions of the world as well as learning from previous lessons. Much of this prior knowledge conflicts with what is currently being taught because of earlier misconceptions.

Persuasive pedagogy is a method of teaching that fosters critical thinking by helping students weigh alternative perspectives, some of those perspectives being their own. It assumes that the student and teacher will both have some understanding of the day's topic, and that only by discussing these understandings will the appropriate and desired learning take place. Persuasive pedagogy starts with the statement of student beliefs and knowledge about a topic. Then the teacher, through the methods of argumentation and justification, will interact with the students to help them understand why the students' beliefs are right or wrong. Both the students and the teacher participate in the discussion of what is "correct" and why. By telling and proving the veracity of the teacher's message, it is assumed the student will not only remember the information for the test, but also most likely remember it forever.

"Teaching as Persuasion" is a relatively new metaphor to describe the most effective practices employed in the act of teaching. This metaphor was developed from the social psychology and persuasive text literatures. Murphy (2001) argued that teachers should take characteristics of the learners and the message, as well as the context of the learning situation, into account when teaching. This must be done, in her view, because learners are diverse and have diverse beliefs about the content at hand. Since teachers must also make sure that students acquire and accept knowledge as correct that is accepted by the scientific community at large, teachers must use instructional methods allowing students the opportunity to change both their knowledge and beliefs concurrently (Chinn and Samarapungavan 2001). By teaching using the more positive tenets of persuasion (Edwards *et al*. 2007; Sinatra and Kardash 2004), teachers will be able to help students change their knowledge and beliefs.

Persuasive pedagogy not only expects students to know about and have experiences with the topic, it actively expects that most of that information will be unexamined and perhaps even incorrect. Using persuasive pedagogical practices requires teachers to create activities in which prior knowledge is both recalled and examined in the learning process. This view of prior knowledge assumes that when the lesson on dividing fractions starts, not only have students learned what a fraction is, but that they have also had experiences in which they have had to divide fractions. This experience should be brought to light in the classroom when dividing fractions is discussed and taught.

Practices of Persuasive Pedagogy in the Mathematics Classroom

Using persuasive practices in a mathematics classroom is different from, and may be somewhat easier than using constructivist practices because there exists a set of well-defined practices encompassing teaching. The hallmark of these is the recognition of and recreation of the interplay of the message and the receiver of the message (Murphy and Alexander 2004). This interplay can be recreated in the classroom by discussion. Discussion should start with an honest appraisal of what is known and believed about the subject. This eliminates the need to infer or guess student knowledge and get straight to the instructional strategies.

It is imperative that mathematics teachers use instructional techniques in these discussions that facilitate student explanations (Billings and Fitzgerald 2002). According to Mason (1998), conceptual understanding can only come through discussion when those teaching techniques comprising persuasive pedagogy are used. In her research in science, she has found that "…a learning classroom context can be a fruitful breeding ground for conceptual understanding when it gives students the opportunity to verbally express their conceptions and explanations, to compare, question, criticize, evaluate them, as well as to write in support of thinking, reasoning, and communicating about knowledge construction" (p. 363). If students do not have the opportunity to uncover their misconceptions through critical reasoning and evaluation of different claims, they will not be able to overcome them.

The types of instructional techniques identified by Mason (1998) have been used extensively in research and literature pertaining to discussions about text. According to Commeyras (1993), for students to be proficient in their thinking, they should evaluate reasons that support multiple conclusions, seek and provide clarification about their own and each other's thinking, and return to the text to settle disputes. In other words, students need to provide some sort of evidence for what they are saying. They might not be able to do these things on their own (Phillips 1995), instead requiring a group to accomplish the tasks of in-depth learning (Commeyras 1993).

Discussion is important for this because student learning is improved when causality is explicit (Chinn and Anderson 1998). Through the use of "interactive argumentation" during a discussion, Chinn and Anderson (1998) state that causal arguments give discussion more organization. This is especially important when we consider that student discussions can sometimes be vague because students do not necessarily say everything they are thinking (Anderson *et al*. 1997, 2007; Noddings 1990). Instead, Anderson and his colleagues found that students might be inclined to say only enough necessary to get their message across.

While these vague discussions can be helpful in increasing student understanding, the mere fact that students do not spontaneously make their reasoning explicit (Anderson *et al.* 1997) can lead to student misconceptions. Teachers must be aware of this and endeavor to become a coach during discussions (Billings and Fitzgerald 2002). They need to teach students how to rely upon observations and facts in making a decision (Lipman 1993) and provide students with ideas for claims they have not considered (Lipman 1993; Waggoner *et al*. 1995). Teachers must encourage students to provide evidence for all the claims they make based upon prior knowledge and the instructional materials they are using (Waggoner *et al*. 1995). They must also encourage students to make their assumptions and thoughts explicit, and model the reasoning process by providing evidence for their own claims (Waggoner *et al*. 1995).

When teachers model their thinking process through discussion and use of evidence in mathematics classrooms and require their students to do the same, misconceptions may be exposed in the same way they are in discussions about text (Anderson *et al*. 1997). Students in mathematics often develop their conceptions based upon inaccurate information (Commeyras 1993) or information that is incomplete (Anderson *et al*. 1997). This uncovering of misconceptions and incomplete understandings through the use of discussion (Anderson *et al*. 1997; Mason 1998) can help students clear up their misunderstandings.

This type of discussion in a mathematics classroom is persuasive in nature because it encourages students to back up their claims with evidence and requires others to judge between claims on the basis of the evidence provided (Buehl *et al*. 2005; Murphy 2001; Murphy and Mason 2006). When students are required to provide evidence for their claims, the discussion will move from a more constructivist approach, or what Keefer *et al*. (2000)

called “explanatory inquiry” into what they termed as “critical discussion.” In explanatory inquiry, students are trying to obtain correct knowledge through discussion with their peers, who may or may not have conceptual understandings. By moving discussion into a critical phase using persuasive practices, students can examine evidence that is provided in order to understand divergent viewpoints. Teachers must facilitate the use of divergent viewpoints and evidence for knowledge gain through persuasive practices (Alexander *et al*. 2002; Mason 1998; Murphy 2001) so that students can determine “why” math works the way it does.

Persuasive Pedagogy and the NCTM Standards

As with constructivism, many of the pedagogical practices associated with persuasive pedagogy are embedded within the NCTM process standards (2000). However, the teaching as persuasion literature gives teachers specific practices for meeting the NCTM standards. Practices comprising a persuasive pedagogy assess students’ initial understandings and beliefs (Alexander *et al*. 2002; Stevens and Fives 2005), and refine insufficient background knowledge using student experiences (Buehl *et al*. 2005). These practices instruct students in the persuasion process (Chinn and Samarapungavan 2001; Hynd 2001), incorporate students’ views in a lesson (Alexander *et al*. 2002), connect instruction to students’ motivation and emotions (Stevens and Fives 2005), and encourage and consider alternative perspectives (Alexander *et al*. 2002). Persuasive practices that can be used to meet the NCTM process standards require students to provide confirming evidence for their positions, recognize multiple sources of authority (Fives and Alexander 2001), analyze the credibility of their sources (Alexander *et al*. 2002), and encourage students to identify problems in arguments (Buehl *et al*. 2005).

Misconceptions in Persuasive Pedagogy

Unlike what is found in constructivist teaching, persuasive pedagogy considers the problem of misconceptions directly. Persuasive pedagogy calls for students to not simply acquire knowledge, but revise existing conceptions and beliefs through deep interaction with content (Buehl *et al*. 2005). This is not directly possible with constructivist teaching because teachers are only able to infer student knowledge and plan experiences according to these inferences, which may initially be wrong. In contrast, persuasive pedagogy is a theory of teaching that builds on what is known about changing people’s opinions through persuasive texts and conceptual change (Murphy 2001; Murphy and Mason 2006; Woods and Demarath 2001). Since much has been written in other articles about the development and viability of this theory (e. g., Edwards *et al*. 2007; Fives and Alexander 2001; Murphy 2001; Sinatra and Kardash 2004), we will only cover highlights that are pertinent to the implementation of this pedagogy in the classroom.

Murphy (2001) describes persuasion as “evoking change in one’s understanding or judgment relative to a particular idea or premise” (p. 224). Past research on persuasive texts informs us that in order to be persuasive, the reader must find the information interesting, personally relevant, and intelligible (Petty and Cacioppo 1986). These same features, plus plausibility and better explanatory power, are necessary if a student is expected to discard prior conceptions in order to accept new ones (Posner *et al*. 1982; Vosniadou 1994). Without these elements, people peripherally process the information, and the message has minimal influence on their beliefs and actions. It follows that these same elements of interestingness, personal relevance, explanatory power, and intelligibility should be part of messages students learn in mathematics classrooms, if the intent is for students to learn and retain the information.

Misconceptions in Mathematics Education

Misconceptions in mathematics are different from those in science, where they are often studied. In math, most of the naïve understandings that children develop before schooling about numeracy are correct. Children learn to recite their numbers, to recognize the ordinality of a set, and to compare quantities at a young age. Children tend to pick up much more knowledge about numbers than they are explicitly taught and it tends to be accurate (Dehaene 1997). Despite this original foundation though, by late elementary school many students are already finding it hard to learn new principles because of the deeply entrenched misconceptions. Most math misconceptions stem from the teaching of mathematics (McNeil and Alibali 2005; Merenluoto and Lehtinen 2004)

Often, math misconceptions seem to be the result of mistaken teaching practices. This is not to say that the math is taught incorrectly; instead, many math procedures are taught with an inadequate or missing conceptual grounding, requiring students to form their own understanding about the underlying concepts and do their own sense-making. Many of these student-created conceptualizations seem sufficient as the student correctly performs the procedure and passes the test. Yet when the concept is used in future mathematics, often the student-formed conceptions may not be appropriately developed, have integrated extraneous details as assumptions, or have missed crucial elements (Moschkovich 1999). These student-developed procedures are likely to either be incorrect or at least nongeneralizable (Wu 1999) and will hinder future learning.

Because math is incremental and each new concept builds on something learned previously, math teachers depend on students having prior knowledge in order to combine their previous conceptions with new factors that allows for a more general conceptual understanding (Merenluoto and Lehtinen 2004). For example, when teaching addition and subtraction of decimals, teachers do not want students to replace all they know about adding and subtracting whole numbers with concepts about adding decimals, but instead to merge the two sets of understanding into a larger concept. Misconceptions complicate this process (Merenluoto and Lehtinen 2004).

Studies of conceptual change often suggest that creating situations of cognitive conflict should help students realize that their previous conceptions are insufficient, and aid in resolving these misconceptions (Chiu 1999). Yet, many students still fail to resolve these misconceptions (Chinn and Brewer 1993; McNeil and Alibali 2005; Merenluoto and Lehtinen 2004). In mathematics, this failure seems to stem from two sources. First, students may not notice that their conceptual understanding is insufficient for the task and proceed forward anyway. A second problem may be overconfidence in their prior knowledge. Their confidence in the validity of their previous ideas is such that though students recognize inconsistencies in their knowledge they choose to retain their prior understandings rather than resolving the inconsistencies (Merenluoto and Lehtinen 2004).

This confidence in prior conceptions is important. In mathematics especially, conceptions created by the students are used exactly because they were found to be successful in understanding and solving earlier problems even when they are inadequate for the problem at hand. Chiu (1996) illustrated this process in an experiment in which students used old solution methods unsuccessfully before turning to a newly taught correct method. Expecting students to immediately accept a method they have had one instructional experience with over prior conceptions that had served them in multiple situations ignores the utility and sense-making a conception supposedly represents. Yet, by not specifically addressing these mistaken or misapplied solution processes, the use of purely constructivist practices may strengthen misconceptions because students are rewarded with "constructing their own knowledge" when they use these prior misconceptions.

Moschkovich (1999) makes an important point for math educators. That is, once a certain level of domain knowledge is acquired, the concept and its underlying assumptions become familiar knowledge. Often, some of the assumptions are tacit. Teachers fail to mention these assumptions because they forget that novices do not understand them. Students though, do not know these assumptions and try to create understandings of the assumptions. As indicated earlier, they may treat what is an extraneous detail as a salient feature of the problem and in trying to construct meaning may lead themselves into misconceptions. Since these misconceptions are developed by the student in a sense-making exercise, the conception is much harder to root out when (and if) it is later exposed.

In fact, McNeil and Alibali (2005) postulated that the old conceptions never quite go away. This theory developed when, after stimulating specific prior addition knowledge in the experimental group, college students were found to make similar mistakes as new learners in problems with operations on both sides of the equation. This finding highlights the need to help students learn correctly. By teaching in a purely constructivist way, misconceptions in mathematics may be strengthened because instead of being discovered and corrected, they are accepted as students' understandings, and then new conceptions are built upon them. Constructivists do not consider the epistemological conditions of whether knowledge is justified or whether those constructed conceptions are even true (Noddings 1990). However, this is not the way mathematics should be taught. While the fact that students take positions and construct their conceptions might work well for more ill structured domains, in mathematics, there is an accepted answer. As Noddings points out, the use of constructivism in mathematics is not for students to come up with an accepted answer to a given problem by their own means, but rather for them to come up with their own answer. Thus, the misconceptions upon which their answers are based may not be discovered.

Persuasive pedagogy not only makes allowances for students to enter the situation with prior knowledge about the topic, it expects it, and finds it essential to the lesson. This is not just in the idea that students have learned what a fraction is, so they are prepared to learn division of fractions, but this is that students have had experiences in which they have had to divide fractions, and that experience should be brought to bear in the classroom as justification when dividing fractions is discussed and taught.

Other Considerations for Implementing Persuasive Pedagogy

Although the promise of increased learning through persuasive pedagogy is strong, there are some additional considerations still to be made. First, a largely ignored but essential factor in a classroom is the role of student beliefs. When beliefs about math are considered, there is a tendency to consider beliefs in ability to perform mathematics, or attitudes about studying mathematics (e.g., math is my favorite subject). These beliefs certainly influence school mathematics performance, yet there are other beliefs that have equally important influences on school, such as a belief that the word "total" in a word problem means to add all the numbers in the problem, or a belief that a parabola only opens in an upward direction.

Though a student would not list these as “beliefs about mathematics,” these beliefs will impact their progress. Teaching using persuasive pedagogy allows teachers to recognize the importance of these beliefs and of changing them because of the role beliefs play in the persuasive process. A discussion of persuasive pedagogy without consideration of beliefs in learning is incomplete.

There are other important considerations in the implementation of persuasive pedagogy. One is the differences between conceptions, procedures, facts, and other types of knowledge, both in formation and alteration (if incorrect). Another is how this paradigm of teaching fits in with current educational policy and recommendations (The National Council of Teachers of Mathematics 2000). Finally, in order to expect teachers to try these practices, it is important to envision how they would be incorporated into regular classroom routines. Without a vision of this, there is no hope for implementation.

The “Monty Hall” Misconception

The contrast between the application of the theoretical framework of persuasive pedagogy and that of constructivism can best be illustrated by example. A famous problem in probability about which students hold misconceptions is referred to as the “Monty Hall Problem” (Selvin 1975a, b). In this problem, Monty Hall offers a contestant the choice of three doors, behind one of which is a car. Behind the other two doors are goats. If the contestant chooses the door behind which is the car, he wins the vehicle. Suppose the contestant chooses door #1, after which Monty tries to urge the contestant to switch doors. Monty then opens up door #3 to reveal a goat and continues to try to persuade the contestant to switch doors. Numerous variations on this problem exist, such as Martin Gardner’s Three Prisoners Problem (Gardner 1959), as well as various conditions under which Monty is or is not required to open a door for the contestant. In this problem, we will assume that Monty is required to open a door regardless of the contestant’s original choice. The question for students is whether it is in the contestant’s best interest to stay with the original door, switch doors, or whether it does not matter. To help students solve this problem, teachers can choose to employ the theoretical frameworks of either constructivism or persuasive pedagogy in their teaching.

For both types of theoretical frameworks, it is hypothesized that teachers would allow students to explore the problem and come up with an answer. Students may be encouraged to make tables, draw pictures, and/or use their logical reasoning skills to determine which choice is in the contestant’s best interest. Teachers would allow their students to draw on their intuition and use that to solve the problem. However, the intuitive answer to the problem is that after Monty reveals door #3, both unopened doors have a 50% chance of disguising a goat (Seymann 1991). An effective teacher would then help students use different methods of solving the problem to confront their erroneous beliefs and determine that it is in the best interest of the contestant to switch doors because door #2 has a two thirds chance of concealing a car.

This example highlights the key difference between constructivism and persuasive pedagogical practices, and shows that the practices effective math teachers intuitively use in the classroom characterize the theoretical framework of persuasive pedagogy rather than that of constructivism. That is, rather than using the theoretical underpinnings of constructivism to help students arrive at the correct answer, effective mathematics teachers would help students confront the beliefs they hold that lead to erroneous conclusions. The adoption of a constructivist framework for teaching necessitates that teachers assume that individuals who explore a particular problem will change their beliefs and modify their alternative conceptions based on the information received and data collected by the exploration. However, students may draw on the appeal to intuition that appears logically correct as the basis for the exploration (Chi 2005). The inclusion of this belief in the exploration would lead to a misconception. Research has shown, however, that individuals' misconceptions are sometimes robust to change and individuals try to hold both the old and new information at the same time (Chinn and Brewer 1993; Vosniadou 1992, 1994). Although this is a possibility with using the practices of persuasive pedagogy, teachers adopting this framework must take an explicit role in helping students correct their underlying beliefs that lead to misconceptions. In contrast, teachers subscribing to a constructivist framework must make the assumption that naïve and incorrect beliefs held by students will be changed once they realize that they no longer function because they believe individuals construct their own realities that are unable to be seen or interpreted by those other than the student. Those subscribing to persuasive pedagogy do not hold such beliefs about student knowledge construction. That is, those who adopt a theoretical framework of persuasive pedagogy believe that it is possible for students to overtly represent their understandings. Those teachers are also able to craft responses to directly confront misconceptions and ill-founded beliefs.

Conclusion

Good mathematics instruction needs the practices promoted by constructivism, such as active experiences with learning, teachers who consider students' capacity while planning lessons, and discussion of mathematical ideas through problem solving. It is vital to convince teachers holding constructivist perspectives that students develop their own understanding through interaction with the concepts. However, there remain some problems in promoting the theoretical framework of constructivism in the classroom.

The first problem comes in the lack of specific practices. Constructivists state that students' mental processes and learning are accentuated by reflecting on their deliberate activities (Phillips 1995). Yet, this learning is not replicable because constructivists do not tell us what those deliberate activities actually are (e.g., Simon 1995). This lack of structure and repeatability could imply that learning rarely occurs. However, it has been recognized that students can learn in well-structured domains, such as science, if specific teaching practices are identified (Mason 1998). Not providing these specific practices implies that to implement constructivism, teachers will find the time to construct their understanding of constructivism and then develop corresponding practices. Without significant changes in the way that schools are run, including provided time to prepare and deeper, more sustained, professional development experiences, the majority of teachers would not be able to accomplish this.

The other problem is the tenet of a lack of an objective reality. Math is built on accepted ideas that are agreed upon as evidence of an objective reality. If students do not understand these ideas, they should be explained and overcome. Yet theoretically, a constructivist teacher cannot label a student's answer as wrong, but only as maladaptive, or an alternate conception. Because constructivist teaching does not accept the notion of an objective reality that can be known it weakens the instructor's ability to directly confront the misconception. As long as a student finds use for a maladaptive conception, it must be treated as viable. Confronting student beliefs leading to misconceptions directly is vital for true learning in mathematics. Contrary to this, some aspects of the domain of mathematics do not leave room for there to always be a second opinion.

Thus, it appears that the use of the current constructivist theory in mathematics education does not prepare students sufficiently for the mathematics that they are being asked to do (Lerman 1996). While the principles of constructivism have value in education, we propose that a reevaluation of the beliefs and principles that underlie instructional strategies is necessary to keep up with the demands of the field. As our understanding of mathematics and learning progresses, it is necessary that our methods of teaching evolve.

The "teaching as persuasion" metaphor adopts many of the instructional strategies practiced by those who ascribe to the constructivist field, yet recognizes the regulations and objectivity inherent in mathematics education. Adopting a theoretical framework of persuasive pedagogy allows the educator the opportunity to examine and help the student change the beliefs underlying their implicit knowledge and construct conceptions in a way that is not theoretically possible with constructivism. Evidence of its use in other domains and the focus on beliefs underlying student knowledge leads us to believe the adoption of the theoretical framework of persuasive pedagogy is the direction in which the change in mathematics education should go.

Persuasive pedagogy practices are valid for mathematics instruction. Persuasive pedagogy provides specific instructional practices that are immediately available for implementation in the classroom. The persuasive teaching practices of discussion, justification, and refutation provide teachers with mechanisms to help students resolve misconceptions.

In the teaching as persuasion metaphor, critical–analytic discussions are employed (Chinn *et al*. 2001). Students are required to justify their answers by providing correct reasons as to why they think what they think, as well as why they employed the procedures they used. Refutation, or providing evidence counter to an incorrect position in support of an opposite position, is also essential to persuasive pedagogical practices to help students overcome misconceptions and to provide students with critical thinking skills.

The most essential part of using persuasion in teaching is having students interact with the topic. In mathematics, this interaction takes place in a process of doing mathematics. Students should have experiences solving problems and developing algorithms for solution. In creating and comparing algorithms in terms of correctness and generalizability, students are placed in a position where they, by examining evidence, discussing alternate views, and providing justification, will hopefully come to a more complete, more correct understanding of mathematics.

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